
REVIEW ON NEWTON RAPHSON METHOD AND APPLICATIONS

Vishal Vaman Mehre^{*1}, Dhruv Rajkumar Sagar^{*2}

^{*1}Assistant Professor, Department Of Electrical Engineering Bharati Vidyapeeth (Deemed To Be University) College Of Engineering, Pune, India.

^{*2}Department Of Electrical Engineering, Bharati Vidyapeeth (Deemed To Be University) College Of Engineering, Pune, India.

ABSTRACT

The Newton method is also known as Newton's Raphson method. It is named after Isaac Newton and Joseph Raphson, two of the most famous scientists of the time. This method is a simple way to obtain an approximation to real-valued roots, as well as to solve non-square and nonlinear problems. It also intends to represent a novel technique to nonlinear equation calculation that is very similar to the Newton Raphson method simple method, with the inverse Jacobian matrix being employed for subsequent calculations and in some applications. The use of a self-derivative function in a scientific calculator to solve non-linear equations, the derivative Newton Raphson formula algorithm, and the uses and limits of the Newton Raphson technique are all explained here.

I. INTRODUCTION

For years, finding the solution to the set of nonlinear equations $f(x) = (f_1, \dots, f_n)' = 0$ has been a challenge. Here, we'll look at this nonlinear equation and see if we can discover a solution using the Newton Raphson approach. The Homotopy method is used to improve the convergence property of numerous methods, and it is well-known for its fast rate convergence [1]. Homotopy works by reducing a difficult problem into a simple one that can then be addressed easily. Further investigation of the subject will necessitate the use of a Homotopy map. In practical applications, root discovery is also an issue. In comparison to other methods, the Newton method is extremely fast and efficient. It is also critical to keep track of the cost and speed of convergence in order to compare performance. Newton method requires only one iteration and the derivative evaluation per iteration. The result of comparing the rate of convergence of Bisection, Complex systems that require faster processing control are in demand these days, and the solution is to divide them into subsystems. This allows each subsystem to be treated separately, with control and operation applied to each component. The research also introduces a new nonlinear distributed load flow calculation technique.

The Newton Raphson method for finding the roots of a nonlinear equation produces good results with a quick convergence speed, and Mat lab has chosen this method for finding the roots. The equipment utilized for such calculations is a scientific calculator. Because they are focused on lowering the interval between two guesses, bracketing methods that require bracketing of the root by two guesses are always convergent [2]. The bracketing method is used in the bisection and false position methods.

NEWTON RAPHSON METHOD

Isaac Newton Joseph Raphson was the inspiration for the Newton Raphson method, which is also known as the Newton method. This is a root-finding technique that provides improved approximations to real-valued roots [3]. It is based on the idea that a tangent to a continuous and differentiable function can approximate it. The aim is to start with a hypothesis that is only close to the true roots, then use mathematics to create a tangent line that intersects the x axis. This procedure can be iterated because the place where the x axis intersects will be a better approximation to the original function root than the first one. By finding a zero in the function's first derivative, any zero-finding method (Bisection Method, False Position Method, Newton-Raphson, etc.) can be used to find a minimum or maximum of such a function; see Newton's method as an optimization algorithm.

The Newton method is derived. $f(x) = 0$

Given a function f defined over the real x and its derivative f' , we begin by guessing x_0 as the root of the function f , then x_1 as the root of the function f'

$x_1 = x_0 - f(x_0)/f'(x_0)$ in graph tangent $(x_0, f(x_0))$ intersect $(x_1, 0)$ at x axis. $x_{n+1} = x_n - f(x_n)/f'(x_n)$ Iteration is continued till the accurate value is reached. [4]

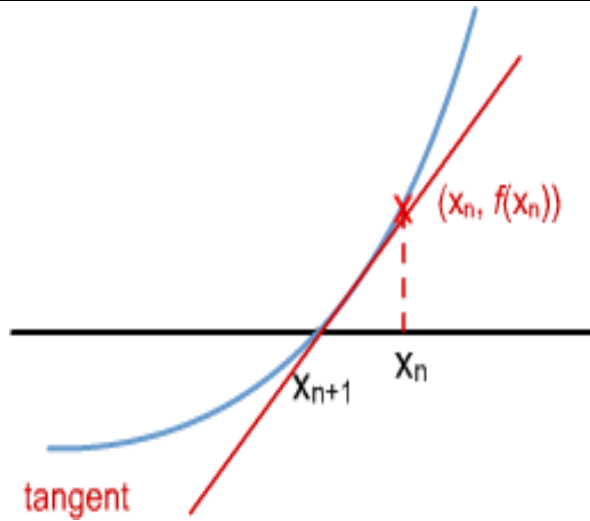


Fig 1.1: Graph illustrating root estimation using Newton Raphson method

Example

Find the real root of $2x^2 - 3\sin x - 6 = 0$ correct up to 4 decimal places with initial value of $x_0 = 1$ using Newton method[5]

$F(x) = 2x^2 - 3\sin x - 6$ $F'(x) = 4x - 3\cos x$ 1st Iteration

$X_0 = 1$

$X_1 = x_0 - f(x_0)/f'(x_0)$ $F(x_0) = -5.424413$

$F'(x_0) = 2.279093$

$X_1 = 1 - (-5.424413)/2.279093$ $X_1 = 3.120668$

2nd iteration $X_2 = 2.327631$

3rd iteration $X_3 = 1.491784$

4th iteration $X_4 = 1.950426$

5th iteration $X_5 = 1.950157$

6th iteration $X_6 = 1.950157$

Root of $f(x) = 1.950157$

Table 1

Number	Iteration
X1	3.120668
X2	2.327631
X3	1.491784
X4	1.950426
X5	1.950157
X6	1.950157

Find the example, if one wishes to find the square root of 602, this is equivalent to finding the solution to [6]

$X^2 = 602$

The function to use in Newton's method is then,

$F(x) = x^2 - 602$

With derivative, $f'(x) = 2x$

With the initial guess of 10, the sequence given by Newton's method is,

1st iteration

$$X1 = x_0 - f(x_0)/f'(x_1)$$

$$= 10 - 10^2 - 602 / 2 * 10$$

$$= 35.6$$

$$\text{2nd iteration } X2 = x_1 - f(x_1)/f'(x_1)$$

$$= 35.6 - 35.6^2 - 602 / 2 * 35.6$$

$$X2 = 26.385555671$$

$$\text{3rd iteration } X3 = 24.890065489$$

$$\text{4th iteration } X4 = 24.738965784$$

5th iteration

$$X5 = 24.73896584$$

Root of f(x) is 24.73896584

Table 2

Number	Iteration
X1	35.6
X2	26.385555671
X3	24.890065489
X4	24.738965784
X5	24.73896584

APPLICATION

- Minimization and maximization problems To find the minimum or maximum of a function f, Newton's technique can be employed (x). [7] Because the derivative is zero at a minimum or maximum, Newton's technique may be used to find local minima and maxima.

The iteration becomes:

$$x_{n+1} = x_n - \frac{f'(x_n)}{f''(x_n)}$$

- Multiplicative inverses of numbers and power Series an important application is Newton–Raphson division, which can be used to quickly find the reciprocal of a number a, using only multiplication and subtraction, that is to say the number x such that 1/x = a. We can rephrase that as finding the zero of f(x) = 1/x - a. We have f'(x) = -1/x².

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n + \frac{\frac{1}{x_n} - a}{\frac{1}{x_n^2}} = x_n(2 - ax_n)$$

- As a result, just two multiplications and one subtraction are required in Newton's iteration.
- The multiplicative inverse of a power series may also be computed quickly using this approach. Solving transcendental equations[8]
 - Many transcendental equations can be solved using Newton's method. Given the equation with g(x) and/or h(x) a transcendental function.
 - The values of x that solve the original equation are then the roots of f(x), which may be found via Newton's method. Obtaining zeros of special functions
 - To get the root of the ratio of Bessel functions, Newton's technique is used.

ADVANTAGES

- Fast convergence: If it converges, it converges quickly. That is, in most circumstances, we can acquire the root (answer) in fewer stages.
- It only takes one guess.

- This strategy is easy to formulate. As a result, it is very simple to implement.
- It has a basic formula that makes it straightforward to programme.
- Derivation is more intuitive, which makes it easier to understand its behavior and predict when it will converge and diverge.[9]

II. LIMITATION

- Convergence isn't a foregone conclusion.
- It is possible to have a division by zero difficulty.
- Root hopping may occur, resulting in the failure to achieve the desired result.
- It's possible that an issue with the inflection point will arise.
- It is necessary to use a symbolic derivative.
- This method converges slowly when there are multiple roots.[10]

III. CONCLUSION

From the research papers, we have concluded that the Newton method's convergence rate is rapid when compared to other approaches. The present injection approach, on the other hand, uses a simple Jacobian matrix and a reduced computation in each iteration, making programming easier and reducing computation time. The Secant method is the most efficient; it has a converging rate similar to that of the Newton Raphson method, but it only requires one function to evaluate per iteration. We also discovered that the bisection method's convergence rate is quite slow, making it impossible to expand such systems equations. As a result, the Newton approach has a quick convergence rate.

IV. REFERENCES

- [1] Saba Akram, Qurrat ul Ann, "Newton Raphson method", International Journal of Scientific & Engineering Research, Volume 6, Issue 7, July 2015'
- [2] Ji Huan He, "A modified Newton Raphson method", Volume 20, Issue 10, 10 June 2004
- [3] Nicholas J Highman, & Hyunmin Kim "Numerical analysis for a quadratic matrix equation", Publication: 5 August 1999 from 13 December 1999
- [4] S.W.Ng & Y.S.Lee, "Variable Dimension Newton Raphson Method", volume no 47, Issue no 6, June 2000
- [5] Waqas Nazeer, Amir Naseem, "Generalized Newton Raphson methods free from second derivative", Published in Journal of Nonlinear Science and Application, April 2016
- [6] Changbum Chun, "Iterative method improving Newton method by the decomposition method", March 2005.
- [7] Rajesh Gupta, Vijaya R. Sawarkar and Pramod R. Bhave "Application of Newton-Raphson method in optimal design of water distribution networks", January 2003.
- [8] Ehiwario J.C, Aghamie S.O, "Comparative Study of Bisection, Newton Raphson and Secant Methods of RootFinding Problems", Volume no 04, Issue no 04, April 2014
- [9] Changbum Chun, "Iterative method improving Newton's method by the decomposition method", March 2005.
- [10] L.R.D.Reis, D.F.Novacki, "The Newton Raphson method in the Extraction of Parameters of Pv modules", April 2017.