

## NON LINEAR AND LINEAR REGRESSION MODELLING OF PRINCIPAL STRESS, STRAIN AND DISPLACEMENT IN A CENTRIFUGAL PUMP IMPELLER DUE TO SPEED VARIATION USING MATLAB

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### ABSTRACT

The study, regression modeling of principal stresses, strain and displacement in a centrifugal pump impeller due to speed variation using MATLAB was successfully carried out. Researchers achieved the study using variable impeller speed, 2 rpm, 100 rpm and 1500 rpm which yielded variable principal stresses, strain and displacement, through finite element analysis approach. The data analysis using MATLAB computed the non linear regression model between 1<sup>st</sup> principal stress and impeller speed. Standard error of the 1<sup>st</sup> principal stress and impeller speed model was observed to be 483.83 with P-value 0.385, degrees of freedom 2 and root mean squared error of 838. The graph suggested that 1<sup>st</sup> principal stress induced in impeller increases with decreasing speed of rotation. Induced strain data also yielded the same non linear regression model with the 1<sup>st</sup> principal stress. Furthermore, linear regression model between impeller speed and 3<sup>rd</sup> principal stress was also generated. The standard error of the model was observed to be 54731, P-value 0.619 with degrees of freedom 1 and root mean squared error of 979. The P-value and degrees of freedom of the generated linear regression model are consistent with the P-value and degrees of freedom of ANOVA model, that proves correctness of the model. The analysis revealed that the impeller speed rotation is in direct/linear relationship with the 3<sup>rd</sup> principal stress. In addition, analysis of impeller speed with displacement data also generated a linear regression model. Standard error was observed to be 3.3459e+07, P-value 0.619 and degrees of freedom 1. The results showed consistence with the P-value and degrees of freedom of Linear model Poly1, that proves correctness of the generated model. The graph indicated that there is an inverse relationship between impeller speed and displacement induced. Researchers made the following recommendations: To avoid failure of centrifugal pump impeller, induced stress and strain due to speed variation must be computed to make appropriate choice of impeller material; Impeller displacement must be computed before assembly is done to ensure appropriate displacement accommodation, etc.

**Keywords:** MATLAB, Regression Model, Principal Stress, Strain, Impeller Speed, Displacement.

### I. INTRODUCTION

#### BACKGROUND OF THE STUDY

Zachwieja (as cited in Olewuezi et al.,2022) stated that the stress, strain, displacement and vibrations in the pipelines can be caused by both the external factors, such as pumps and the internal factors like pulsating motion of the fluid and fluid weight. The vibrations may result in immediate fatigue damage as a long-term effect to fluid pumping system.

Impeller vanes are important components of a centrifugal pump and therefore, require stress and displacement analyses if efficiency and performance must be optimized. The main requirement is to ensure that the impeller blade or vanes can withstand the level of stresses, strains and displacement that manifest during pump operation thereby reducing the chance of blade fracture/failure (Mohamed, Moey, Ibrahim, Yazdi, & Merdji, 2022).

Rajput (2008) stated that the centrifugal pump is a contrivance which provides energy to a fluid in a fluid system; it assists to increase the pressure energy or kinetic energy, or both of the fluid by converting the mechanical energy of a prime mover or electric motor coupled to its shaft. An impeller is a wheel a series of backward curved vanes or blades and it is mounted on a shaft which is usually coupled to an electric motor.

Khurmi and Gupta (2014) explained that the principal stresses are maximum and minimum value of normal stresses on a plane, when rotated through an angle on which there is no shear stress. The principal plane is that plane the principal stresses act and shear stress is zero. 1<sup>st</sup> principal stress represents stress due to tensile loading whereas 3<sup>rd</sup> principal stress represents stress due to compressive loading. Stress induces strain and displacement.

Regression modeling here is a non linear and linear approach for modeling the relationship between a dependent variables stress, strain or displacement and one independent variable pump speed.

Obviously, there are induced stresses, strains and displacement in impeller blades due to pump speed variation. Hence, the paper aimed at studying regression modeling of principal stresses, strain and displacement in a centrifugal pump impeller due to speed variation using MATLAB.

### **STATEMENT OF PROBLEM**

It is of paramount to research on the stress, strain, displacement and vibration characteristics of the pump impeller since it influences pump reliability, stability and pump efficiency.

Impeller vanes are important components of a centrifugal pump and therefore, require stress and displacement analyses if efficiency and performance must be optimized. The main requirement is to ensure that the impeller blade or vanes can withstand the level of stresses, strains and displacement that manifest during pump operation thereby reducing the chance of blade fracture/failure (Mohamed, Moey, Ibrahim, Yazdi, & Merdji , 2022). It is on this note that the researchers aimed at determining the regression modeling of principal stresses, strains and displacement in a centrifugal pump impeller due to speed variation using MATLAB.

### **PURPOSE OF THE STUDY**

The general purpose of the study is to determine the linear and non linear regression models of induced principal stresses, strains and displacement in a centrifugal pump impeller due to speed variation using MATLAB.

### **SIGNIFICANCE OF THE STUDY**

The result of this study will be beneficial to industrial pump designers/production engineers in the following ways:

- 1) Production engineers can improve fluid flow safety; reduce flow noise and avoid sudden pump failure by choosing pump material whose allowable stress is greater than induced stress due to speed variation.
- 2) The knowledge of displacement can be used to improve the design life of pumps by making appropriate provisions for expansion within pump casing.

### **SCOPE OF THE STUDY**

Researchers will focus on establishing the linear and non linear regression models between the induced principal stresses, strains and displacement due to speed variation. So, all efforts will be directed towards the general objectives. Stress, strain and displacement data evaluations followed Finite Element Analysis approach and is beyond the scope of this paper. The researchers are members of Federal Polytechnic Nekede and Gregory University Uтуру, within South East of Nigeria. Results may be subject to variations within other parts of the World.

## **II. LITERATURE REVIEW**

Olewuezi et al. (2022) studied linear regression modeling of induced stress and displacement along a pipeline due to fluid weight using matlab approach. They concluded that to avoid failure and fluid leakage in pipelines, induced stress due to fluid weight must be computed to make appropriate choice of pipe material and Pipeline displacement must be computed before pipe joining is done to ensure appropriate displacement accommodation. Mohamed et al,( 2022) evaluated Stress Analysis of Various Designs of Centrifugal Pump Impellers Using Finite Element Method. They concluded that the impeller requires stress and displacement analyses if efficiency must be maximized. The main requirement is to ensure that the impeller blade can withstand the level of stress thereby reducing the chance of blade fracture/failure. Rajput (2008) studied Fluid Mechanics and Hydraulic Machines and stated that the centrifugal pump is a contrivance which provides energy to a fluid in a fluid system; it assists to increase the pressure energy or kinetic energy, or both of the fluid by

converting the mechanical energy of a prime mover or electric motor coupled to its shaft. Khurmi and Gupta (2014) studied Strength of Materials and explained that the principal stresses are maximum and minimum value of normal stresses on a plane, when rotated through an angle on which there is no shear stress. The principal plane is that plane the principal stresses act and shear stress is zero. 1<sup>st</sup> principal stress represents stress due to tensile loading whereas 3<sup>rd</sup> principal stress represents stress due to compressive loading. Stress induces strain and displacement.

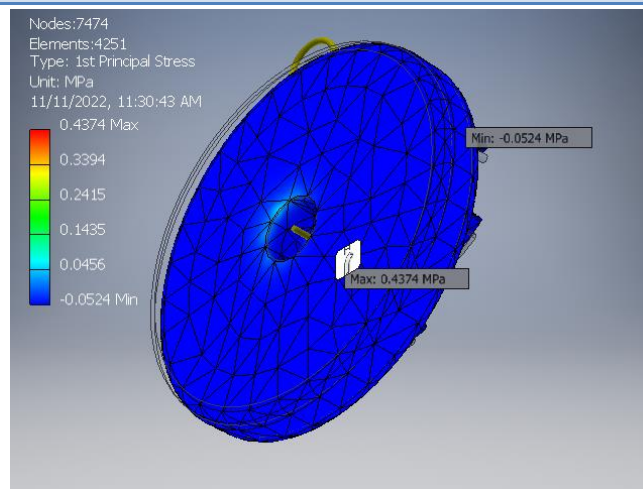
### III. METHODOLOGY

The researchers considered variable pump speed, which yielded different principal stresses, strains and displacement as shown in **table 1.0**. Finite Element Analysis was used to evaluate principal stresses, strain and displacements induced due to variable pump speed (See **figure 1.0 -1.3**).

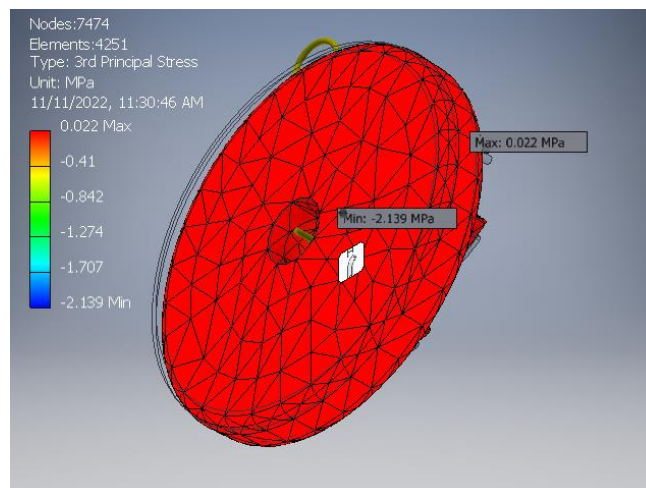
### IV. RESULTS AND PRESENTATIONS

**Table 1.0:** shows maximum principal stresses, strains and maximum displacements due to speed variation.

| S/N | Speed(rpm) | Maximum 1 <sup>st</sup> Principal Stress (MPa) | Maximum 3 <sup>rd</sup> Principal Stress (MPa) | Maximum Strain        | Maximum Displacement(mm) |
|-----|------------|--|--|-----------------------|--------------------------|
| 1   | 2          | 0.437364                                       | 0.0221447                                      | 0.00000494271         | 0.0000362235             |
| 2   | 100        | 0.008747                                       | 0.000443                                       | $9.89 \times 10^{-8}$ | 0.00000072447            |
| 3   | 1500       | 0.000583                                       | 0.0000295                                      | $6.59 \times 10^{-9}$ | 0.0000000483             |



**Figure 1.0:** 1st Principal Stress



**Figure 1.1:** 3<sup>rd</sup> Principal Stress

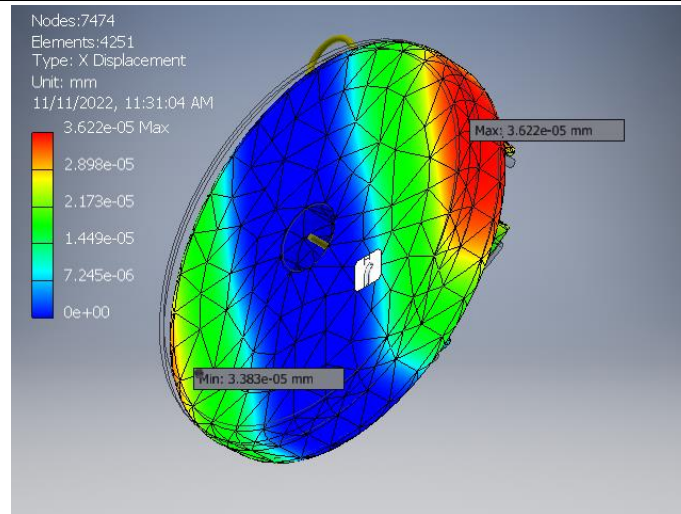


Figure 1.2: Displacement

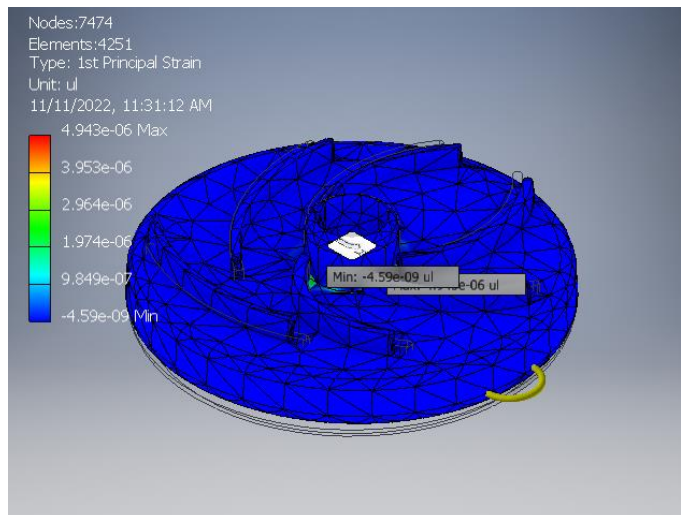


Figure 1.3: 1st Principal Strain

>> %MATLAB PROGRAMME FOR SPEED VARIATION AND 1ST PRINCIPAL STRESS

>> % Y = PUMP SPEED IN RPM

>> % X = 1ST PRINCIPAL STRESS IN MPa

>> Y = [2 100 1500];

>> X = [0.437364 0.008747 0.000583];

>> modelfun = @(b,x)b(1) + b(2)\*x(:,1).^b(3);

beta0 = [2 100 1500];

mdl = fitnlm(X,Y,modelfun,beta0)

mdl =

Nonlinear regression model:

$$y \sim b1 + b2*x1^b3$$

Estimated Coefficients:

| Estimate | SE | tStat | pValue |
|----------|----|-------|--------|
|----------|----|-------|--------|

|    |     |        |        |         |
|----|-----|--------|--------|---------|
| b1 | 534 | 483.83 | 1.1037 | 0.38476 |
|----|-----|--------|--------|---------|

|    |     |   |     |   |
|----|-----|---|-----|---|
| b2 | 100 | 0 | Inf | 0 |
|----|-----|---|-----|---|

b3 1500 0 Inf 0

Number of observations: 3, Error degrees of freedom: 2

Root Mean Squared Error: 838

R-Squared: 0, Adjusted R-Squared 0

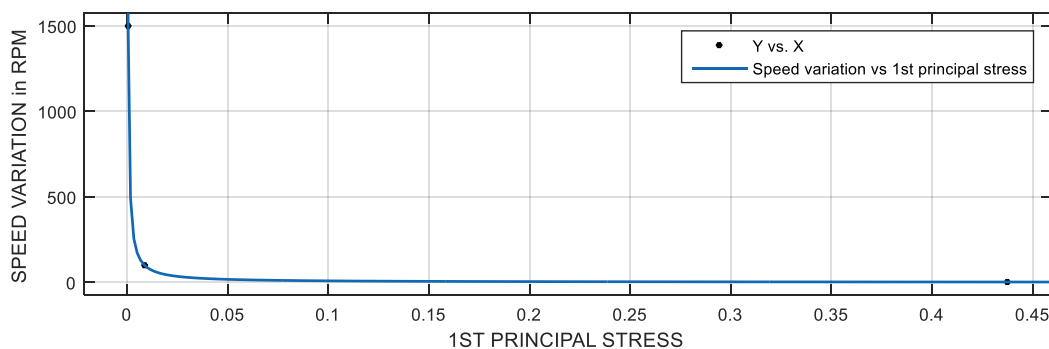
F-statistic vs. zero model: 1.22, p-value = 0.385

>> end

The computed non linear regression model between speed variation and 1st principal stress is shown below;

$$Y = 534 + 100X^{1500}$$

Where Y = Speed variation in rpm and X = 1st Principal Stress in MPa.



**Fig 1.0:** Graph of Impeller Speed Variation against 1<sup>st</sup> Principal Stress.

MATLAB CODES/SCRIPTS OF THE GRAPH ABOVE

```
function createfigure1(XData1, YData1, XData2, YData2)
%CREATEFIGURE1(XDATA1, YDATA1, XDATA2, YDATA2)
% XDATA1: line xdata
% YDATA1: line ydata
% XDATA2: line xdata
% YDATA2: line ydata
% MATLAB on 23-Nov-2022 11:01:26
% Create figure
figure1 = figure('Tag','Print CFTOOL to Figure',...
'Color',[0.941176470588235 0.941176470588235 0.941176470588235]);
% Create axes
axes1 = axes('Parent',figure1,'Tag','sftool surface axes');
%% Uncomment the following line to preserve the X-limits of the axes
% xlim(axes1,[-0.02126 0.45921]);
%% Uncomment the following line to preserve the Y-limits of the axes
% ylim(axes1,[-72.9 1574.9]);
%% Uncomment the following line to preserve the Z-limits of the axes
% zlim(axes1,[-1 1]);
box(axes1,'on');
grid(axes1,'on');
hold(axes1,'on');
% Create line
line(XData1,YData1,'Parent',axes1,'DisplayName','Y vs. X',...
'MarkerFaceColor',[0 0 0],...
```

```
'MarkerEdgeColor',[0 0 0],...
'MarkerSize',3,...
'Marker','o',...
'LineStyle','none');
% Create line
line(XData2,YData2,'Parent',axes1,...
'DisplayName','Speed variation vs 1st principal stress',...
'LineWidth',1.5,...
'Color',[0.0705882352941176 0.407843137254902 0.701960784313725]);
% Create xlabel
xlabel({'1ST PRINCIPAL STRESS'});
% Create ylabel
ylabel({'SPEED VARIATION in RPM'});
% Create zlabel
zlabel('Z');
% Create legend
legend1 = legend(axes1,'show');
set(legend1,'Interpreter','none','EdgeColor',[0.15 0.15 0.15]);
>> % MATLAB PROGRAMME FOR SPEED VARIATION AND 3RD PRINCIPAL STRESS
>> % A = PUMP SPEED IN RPM
>> % B = 3RD PRINCIPAL STRESS IN MPa
>> A = [2 100 1500];
>> B = [0.0221447 0.000443 0.0000295];
>> modelfun = @(b,x)b(1) + b(2)*x(:,1).^b(3);
beta0 = [2 100 1500];
mdl = fitnlm(A,B,modelfun,beta0)
Error using nlinfit>checkFunVals (line 636)
The function you provided as the MODELFUN input has returned Inf or NaN values.
>> mdl = fitlm(B,A)
mdl =
Linear regression model:
y ~ 1 + x1
Estimated Coefficients:
      Estimate      SE      tStat      pValue
      _____  _____  _____  _____

(Intercept) 815.35   699.89    1.165   0.45158
x1          -37319   54731   -0.68186 0.61902
Number of observations: 3, Error degrees of freedom: 1
Root Mean Squared Error: 979
R-squared: 0.317, Adjusted R-Squared -0.365
F-statistic vs. constant model: 0.465, p-value = 0.619
>> mdl1 = stepwiselm(mdl)
>> tbl = anova(mdl)
```

tbl =

|       | SumSq      | DF | MeanSq     | F       | pValue  |
|-------|------------|----|------------|---------|---------|
| x1    | 4.4576e+05 | 1  | 4.4576e+05 | 0.46493 | 0.61902 |
| Error | 9.5877e+05 | 1  | 9.5877e+05 |         |         |

The computed linear regression model between speed variation and 3rd principal stress is shown below;

$$Y = -37319X + 815.35$$

Where Y = Speed variation in rpm and X = 3rd Principal Stress in MPa.

**>> %MATLAB PROGRAMME FOR SPEED VARIATION WITH STRAIN**

>> % D = PUMP SPEED IN RPM

>> % C = STARIN

>> D = [2 100 1500];

>> C = [0.00000494271 0.0000000989 0.00000000659];

>> modelfun = @(b,x)b(1) + b(2)\*x(:,1).^b(3);

beta0 = [2 100 1500];

mdl = fitnlm(C,D,modelfun,beta0)

mdl =

Nonlinear regression model:

$$y \sim b1 + b2*x1^b3$$

Estimated Coefficients:

|    | Estimate | SE     | tStat  | pValue  |
|----|----------|--------|--------|---------|
| b1 | 534      | 483.83 | 1.1037 | 0.38476 |
| b2 | 100      | 0      | Inf    | 0       |
| b3 | 1500     | 0      | Inf    | 0       |

Number of observations: 3, Error degrees of freedom: 2

Root Mean Squared Error: 838

R-Squared: 0, Adjusted R-Squared 0

F-statistic vs. zero model: 1.22, p-value = 0.385

>>end

The computed non linear regression model between speed variation and strain is shown below;

$$Y = 534 + 100X^{1500}$$

Where Y = Speed variation in rpm and X =Strain.

**>> % MATLAB PROGRAMME FOR SPEED VARIATION WITH DISPLACEMENT**

>> % E = PUMP SPEED in RPM

>> % F = DISPLACEMENT in mm

>> E = [2 100 1500];

>> F = [0.0000362235 0.00000072447 0.0000000483];

>> mdl = fitlm(F,E)

mdl =

Linear regression model:

$$y \sim 1 + x1$$

Estimated Coefficients:

|             | Estimate    | SE         | tStat    | pValue  |
|-------------|-------------|------------|----------|---------|
| (Intercept) | 815.35      | 699.89     | 1.165    | 0.45159 |
| x1          | -2.2814e+07 | 3.3459e+07 | -0.68185 | 0.61902 |

Number of observations: 3, Error degrees of freedom: 1

Root Mean Squared Error: 979

R-squared: 0.317, Adjusted R-Squared -0.365

F-statistic vs. constant model: 0.465, p-value = 0.619

>>end

The computed linear regression model between speed variation and displacement is shown below;

$$Y = -2.2814 \times 10^7 X + 815.35$$

**LINEAR MODEL POLY1:**

$$f(x) = p1*x + p2$$

Coefficients (with 95% confidence bounds):

$$p1 = -2.281e+07 \quad (-4.48e+08, 4.023e+08)$$

$$p2 = 815.3 \quad (-8078, 9708)$$

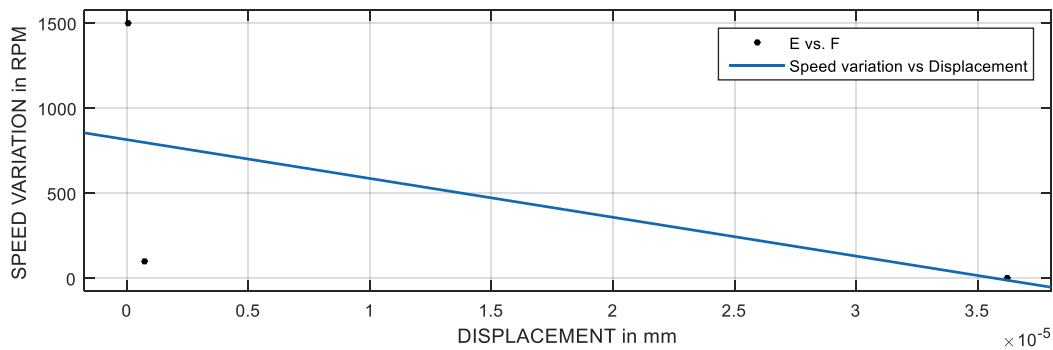
Goodness of fit:

SSE: 9.588e+05

R-square: 0.3174

Adjusted R-square: -0.3653

RMSE: 979.2 here Y = Speed variation in rpm and X = Displacement in mm.



**Fig 1.1:** Graph of Impeller Speed Variation against Displacement.

MATLAB CODES/SCRIPTS OF THE GRAPH ABOVE

```
function createfigure1(XData1, YData1, XData2, YData2)
%CREATEFIGURE1(XDATA1, YDATA1, XDATA2, YDATA2)
% XDATA1: line xdata
% YDATA1: line ydata
% XDATA2: line xdata
% YDATA2: line ydata
% Auto-generated by MATLAB on 23-Nov-2022 11:10:38
% Create figure
figure1 = figure('Tag','Print CFTOOL to Figure',...
    'Color',[0.941176470588235 0.941176470588235 0.941176470588235]);
% Create axes
```



```
axes1 = axes('Parent',figure1,'Tag','sftool surface axes');
%% Uncomment the following line to preserve the X-limits of the axes
% xlim(axes1,[-1.761e-06 3.8033e-05]);
%% Uncomment the following line to preserve the Y-limits of the axes
% ylim(axes1,[-72.9 1574.9]);
%% Uncomment the following line to preserve the Z-limits of the axes
% zlim(axes1,[-1 1]);
box(axes1,'on');
grid(axes1,'on');
hold(axes1,'on');
% Create line
line(XData1,YData1,'Parent',axes1,'DisplayName','E vs. F',...
'MarkerFaceColor',[0 0 0],...
'MarkerEdgeColor',[0 0 0],...
'MarkerSize',3,...
'Marker','o',...
'LineStyle','none');
% Create line
line(XData2,YData2,'Parent',axes1,...
'DisplayName','Speed variation vs Displacement',...
'LineWidth',1.5,...
'Color',[0.0705882352941176 0.407843137254902 0.701960784313725]);
% Create xlabel
xlabel({'DISPLACEMENT in mm'});
% Create ylabel
ylabel({'SPEED VARIATION in RPM'});
% Create zlabel
zlabel('Z');
% Create legend
legend1 = legend(axes1,'show');
set(legend1,'Interpreter','none','EdgeColor',[0.15 0.15 0.15]);
```

## V. DISCUSSION

The results of the study, regression modeling of principal stresses, strain and displacement in a centrifugal pump impeller due to speed variation using MATLAB were discussed here. Researchers achieved the study using variable impeller speed, 2 rpm, 100 rpm and 1500 rpm which yielded variable principal stresses, strain and displacement as shown in **table 1.0**.

The MATLAB analysis also computed the non linear regression model between 1<sup>st</sup> principal stress and speed variation;

$$\text{Speed (rpm)} = 534 + 100(\text{1st Principal Stress in MPa})^{1500}$$

The standard error was observed to be 483.83. The P-value 0.385 and degrees of freedom 2 with root mean squared error of 838. The graph of **Fig. 1.0**, suggested that 1<sup>st</sup> principal stress induced in impeller increases with decreasing speed of rotation.

Computed linear regression model between impeller speed variation and 3<sup>rd</sup> principal stress is shown below;

$$\text{Speed (rpm)} = -37319 \times \text{3rd Principal Stress in MPa} + 815.35$$

The standard error was observed to be 54731. The P-value 0.619 and degrees of freedom 1 with root mean squared error of 979. The P-value and degrees of freedom of the generated linear regression model are consistent with the P-value and degrees of freedom of ANOVA model, that proves correctness of the model. The analysis revealed that the impeller speed rotation is in direct/linear relationship with the 3<sup>rd</sup> principal stress.

Non linear regression model between impeller speed variation and strain is shown below;

$$\text{Speed (rpm)} = 534 + 100(\text{Strain})^{1500}$$

This indicated that strain induced in impeller increases with decreasing speed of rotation, in line with the 1<sup>st</sup> principal stress.

Displacement also followed a linear regression modeling approach and is as shown below;

$$\text{Speed (rpm)} = -2.2814 \times 10^7 \times \text{Displacement} + 815.35$$

The standard error was observed to be 3.3459e+07. The P-value 0.619 and degrees of freedom 1 of the linear regression model generated are consistent with the P-value and degrees of freedom of Linear model Poly1, that proves correctness of the generated model.

## VI. CONCLUSION

The regression modeling of principal stresses, strain and displacement in a centrifugal pump impeller due to speed variation using matlab was obviously achieved. Undoubtedly, results revealed that at any given value of impeller speed, induced principal stresses, strain and displacement can be easily computed. This study is also in line with the induced stress and displacement study conducted by Olewuezi et al, 2022.

## VII. RECOMMENDATIONS

The following recommendations are suggested based on the study:

- 1) To avoid failure of centrifugal pump impeller, induced stress and strain due to speed variation must be computed to make appropriate choice of impeller material.
- 2) Impeller displacement must be computed before assembly is done to ensure appropriate displacement accommodation.
- 3) This research can also be done using other advanced software for generalization.

## VIII. REFERENCES

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