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DESIGN OF A MINIMUM LENGTH SUPER SONIC NOZZLE

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ABSTRACT

A Super Sonic Nozzle is a duct that is compressed in the middle, forming a convergent-divergent shape like an hourglass. It accelerates flow to supersonic speeds, by exchanging the pressure and thermal energy into the kinetic energy. Super Sonic nozzles are widely employed in some types of steam turbines, supersonic jet engines, and rocket engine nozzles.

Supersonic nozzle should have a shock-free and parallel steady flow. The substantial conditions of an ideal 2-dimensional, unfaltering stream may be communicated numerically by using the nonlinear differential condition of the potential flow. The strategy of characteristics may be a numerical detailing that could be utilized to dig up to the answers to the previously mentioned potential flow, fulfilling the boundary conditions so the overseeing differential equations become ordinary differential equations.

A scheme of the method of characteristics is utilized here for 2-dimensional, C-D nozzle design. In applications like a rocket engine and supersonic aircraft, the length of the nozzle cannot be too large for less weight requirement, so we need to have a least possible length of the C-D nozzle. Hence, the method of designing a Minimum length C-D nozzle, for the designed exit Mach number with the steady flow at the diverging part of the nozzle, is presented in this thesis.

I. INTRODUCTION

THEORETICAL BACKGROUND

The Nozzle

An appliance which is used to oversee the characteristics of a stream to enhance its speed as it quits an encased boudoir is well known as **Nozzle**.

It could be a duct of varying cross-sectional zone and can be utilized to coordinate or adjust the stream. Nozzles are as often as possible utilized to accelerate the flow of the working fluid. The speed of the flow increases due to the conversion of pressure into the kinetic energy.

Nozzle's implementations are**:**

1. They are employed in jet turbines, steam-based turbines.

2. They are also employed in flow measurement e.g., in venturi meter.

3. They are employed to extract air out of condenser.

Here I will be discussing about propelling Nozzles only i.e., Rocket engine nozzle and Aircraft nozzle. These type of Nozzle converts available Energy of the fluid into a high-speed propelling jet.

The nozzles which are used for propulsion purpose expands the velocities to the supersonic, transonic or sub sonic speeds depending upon the type of it.

The inner geometry of the nozzle may be decreasing area or first decreasing area than increasing area. If it is only decreasing area than it is a convergent one or if it is the second case than it is a convergent divergent nozzle. The first one operates up to Mach 1 and the later one is mainly employed for Mach greater than 1.

The geometry of the nozzle can be made variable by using the arrangements. The variable geometry nozzles provide the variation in the operating envelope. These are employed in after burning jet engines where there is requirement to have a variable throat and exit area.

Types of Nozzles

1) Convergent
2) De Level (Supersonic nozzle)

The Convergent Nozzle:

Convergent nozzle begins enormous and gets smaller-a diminish in cross-sectional zone towards the conclusion. As flow starts at the beginning and proceeds towards the end where the area is progressively getting smaller, the velocity of the flow will increase to validate the conservation of mass rule. So, the mass flow will be constant throughout the duct and it will cause the pressure to decrease. Pressure drop in a convergent nozzle is explained with the help of the Bernoulli's Principle.

Convergent Nozzle can accelerate fluid up to sonic speed (M=1) i.e., up to speed of sound. A Figure of Convergent nozzle is shown below (Figure 1).

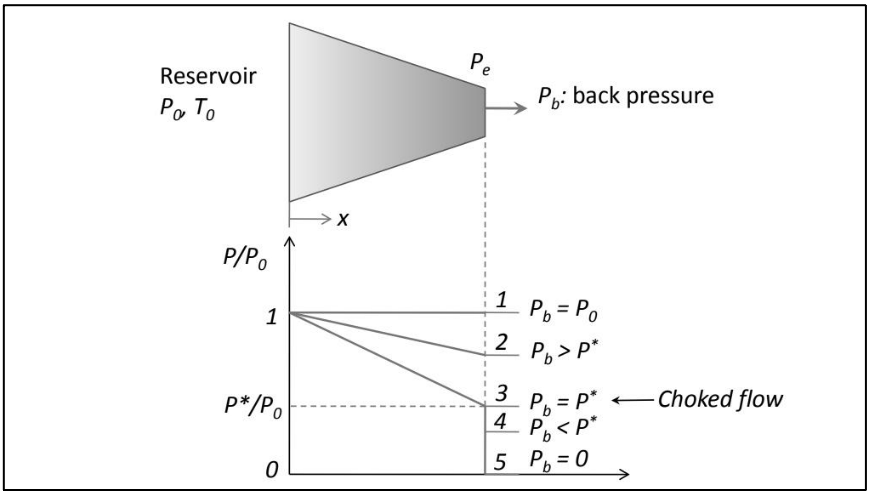

Figure 1: A Convergent Nozzle operation

The Convergent Divergent Nozzle:

A convergent divergent nozzle is a duct or any other shape that is compressed in between to make an hourglass kind of shape. The principal use is to generate the supersonic flow. It was designed and developed by a Swedish inventor 'Gustaf de Laval' in 1888 to use on a steam-turbine.

The flow is at low subsonic speed, when it enters a C-D nozzle. As we know that the area continuously decreases towards the throat and it again increases towards the end from the throat. The velocity increases in both the part of the c-d nozzle. In divergent part it is a supersonic expansion where the area also increases with the velocity, in the convergent part the velocity increases due to the Bernoulli's principle. Figure 2 shows the working of a supersonic nozzle.

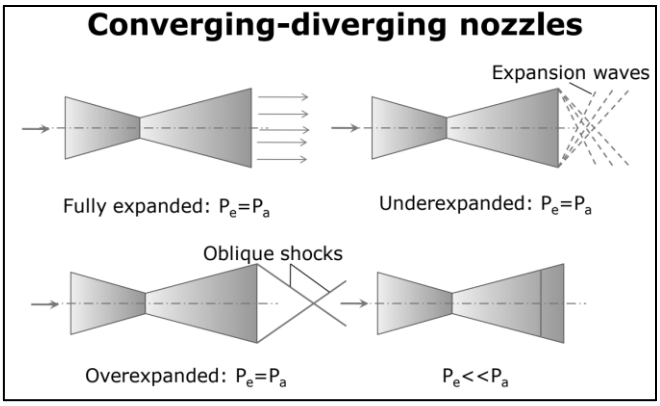

Figure 2: Convergent Divergent Nozzle Operation

II. METHODOLOGY

A brief introduction of the method which is used to perform the analysis is documented in this section.

Thermodynamic Equations:

The linear velocity and other thermodynamic properties of the outgoing fluid for isentropic scenario is calculated using the following equations:

Conservation of mass:

$$m = \rho * A * V = \text{Constant}$$

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

Conservation of The Momentum:

$$* \; V \; * \; dV = - dp$$

Conservation of Energy:

$$m * h_0 = m * h_{exit} + (m * v_{exit}^{\;2}) * 0.5$$

Isentropic Relations:

The fundamental equation for an ideal gas is

P * volume / T = constant = R

Knowing that ν = Volume / m

P * Volume / T * m = constant = R

The R is a gas constant of the fluid

The important relation derived from the above equations is frequently used to calculate the Mach number of flows,

$$T_0 / T = (1 + (\gamma - 1) * 0.5 * Mach^2)$$

Basics Method of Characteristics:

Characteristics are 'lines' in a super-sonic stream situated in a particular direction along which unsettling influences are proliferated. It is a numerical technique employed to solve 2-dimensional compressible flow problems among other things. Using this technique flow properties are calculated at distinct points.

Characteristic's properties are:

1. In a 2-dimensional supersonic stream it may be a line or a bent along which physical unsettling influences are engendered at the regional speed of sound relative to the gas.
2. It could be a bent over which stream properties are ceaseless, in spite of the fact that they may have spasmodic, to begin with, subsidiaries, and along which the subsidiaries are uncertain.
3. It is a bend along which the governing pde could be manipulated into an ode.

The characteristics are turned from it's course at the throat and it make and expansion fan of a series of waves. This phenomenon of the characteristics waves is typically modelled as a continuous series of expansion waves. So, it turns the flow an infinitesimal amount along with the curve of the wall. These waves could be the opposite of shock waves, which slows down the flow. It is given by the Prandtl-Meyer function [1].

$$\nu \; (Mach) = \sqrt{\frac{\Upsilon+1}{\Upsilon-1}}\, tan^{-1} \sqrt{\frac{\Upsilon-1}{\Upsilon+1}(\,Mach^2 \;-\; 1)} - tan^{-1} \; \sqrt{(Mach^2 \;- 1\,)}$$

The analysis in the work presented here uses the following equations; the angle of the stream with respect to the horizontal plain is represented by the symbol 'ν'. The Mach angle μ is defined as:

$$\mu = \text{arc sin } (1 / M)$$

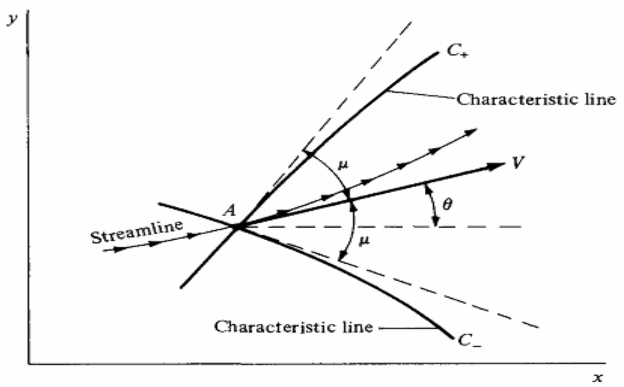

Figure 3: Characteristic Lines

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Θ + ν (Mach) = constant = K- (beside C- characteristic)

Θ - ν (Mach)= constant = K+ (beside C+ characteristic)

The stream in the super-sonic region can be divide into two regions namely simple and non simple regions. Mach wave reflections and crossing points represents a non simple region

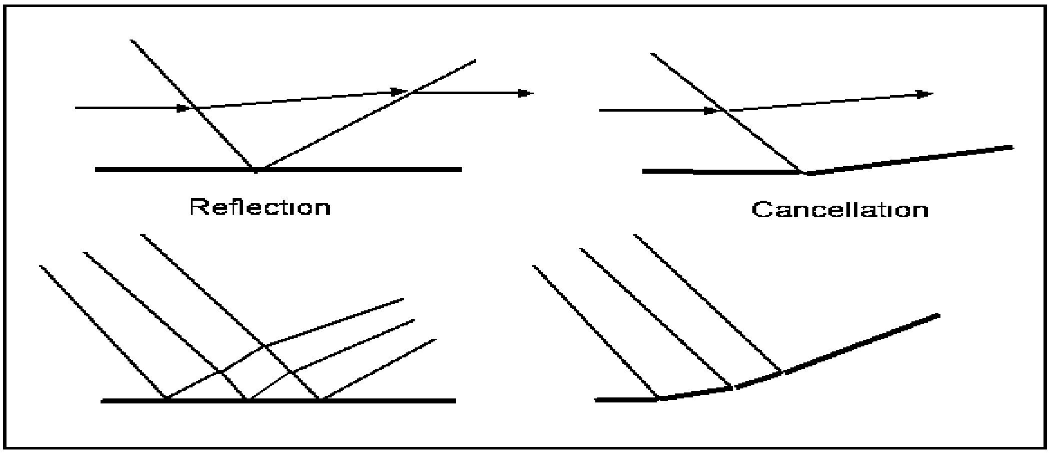

Figure 4: Shock Wave Interaction

In arrange to extend an inner consistent (steady) stream from a channel from sub-sonic to super-sonic speed, the channel has got to be convergent divergent in shape. In the event that the nozzle form isn't legitimate, shock waves may happen interior of the conduit. The strategy of characteristics gives a strategy for appropriately planning the form of a super-sonic Nozzle for the shock free, isentropic stream, taking into consideration the multi-dimensional stream interior of the conduit.

The aim of this section is to highlight the usefulness of the mentioned method.

Moreover, in cases where fast expansions are alluring, such as the non equilibrium stream in advanced gas energetic lasers, the nozzle length is as brief as conceivable. In such a smallest length nozzle, the extension segment is contracted to a point, and the extension takes put through a centered **Prandtl Meyer** wave radiating from a sharp corner throat with a point $\theta Wmax$ as outlined in Figure 5. The length of the super-sonic nozzle signified as L in Figure is the least esteem reliable with the shock free, isentropic stream. In the event that the form is made smaller than L, shocks will be created in the nozzle [1].

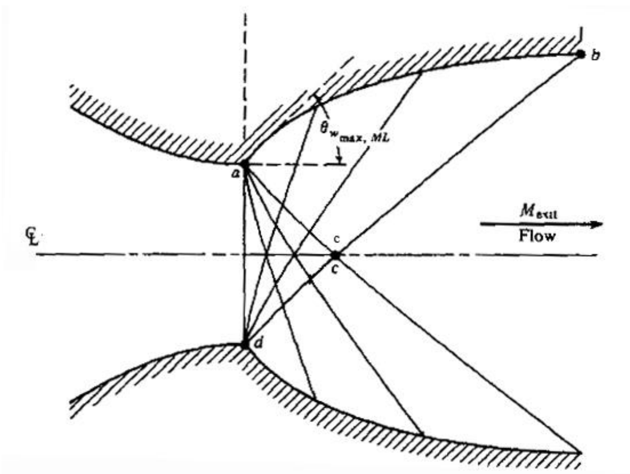

Figure 5: MOC Approach

Supersonic Nozzle Design:

As discussed earlier the Supersonic nozzle consist of two sections one is convergent and the other is divergent. The MOC approach is mainly used in the design of a divergent part of the nozzle as it has super-sonic flow and shock waves can only form in this section, while the convergent part is totally a subsonic region.

Here the design of the supersonic nozzle is explained in two parts. First, we will discuss convergent part and then the divergent part.

The objective is to design an efficient C-D nozzle. The following “general” criteria are desired in the design.

Stagnation pressure at Inlet (P_t) = 10 bar

Stagnation temperature at Inlet (T_t) = 300°C

Mach number Inlet= 0.1

Mach number Exit= 2.0

Mass flow rate (ṁ) = 8 kg/s

Convergent Section Design:

In this section, we will discuss the design of a convergent nozzle according to the design objectives mentioned above.

For analysis the nozzle geometry is considered as rectangular with unit width, so the area at a particular section will be height multiplied by width (which is unit width) at the particular section.

Area = height * width

As we assumed unit width:

Area = height

To calculate the area at each section we need to have a throat area, throat area is calculated as follows:

$$\frac{P_0}{P_t} = \left[1 + \frac{\Upsilon-1}{2} M^2\right]^{\frac{\Upsilon-1}{\Upsilon}} \quad \Rightarrow \quad P_t = 4.2262 \text{ bar}$$

$$\frac{T_0}{T_t} = \left[1 + \frac{\Upsilon-1}{2} M^2\right] \quad \Rightarrow \quad T_t = 302.7055 \text{ K}$$

$$\rho_t = \frac{p_t}{RT_t} = 4.8616 \text{ kg}/m^3$$

$$V_t = \sqrt{\Upsilon RT_t} = 348.86 \text{ m/s}$$

$$\text{Mass flow rate } (\dot{m}) = p_t A_t T_t$$

$$\text{Throat Area } (A_t) = \dot{m} / p_t T_t = 62.53\ cm^2$$

Now the area Mach relation is used to get area at each station:

$$\frac{A}{A_t} = \frac{M_t}{M}\left[\frac{1 + \frac{\Upsilon-1}{2} M^2}{1 + \frac{\Upsilon-1}{2} M_t}\right]^{\frac{\Upsilon}{\Upsilon-1} - \frac{1}{2}}$$

We will start at the inlet and calculate the area at each increase of 0.01 in Mach number, For example at the inlet area will be given by the above equation is:

$$A_{(M=0.1)} = 364.06 \text{ cm}^2$$

We have assumed unit width for a rectangular cross section of the nozzle, hence the height (H) at the entry will be:

H = 364.06 cm and H/2 = 182.03 cm

Similarly, the data for each station is calculated and tabled.

The data from the table is plotted and shown in the below Figure. The shape variation is sharp at the beginning and it slows down at the end. The above table has presented data about the height of the nozzle only, the details about the length of the nozzle will be discussed in the future work.

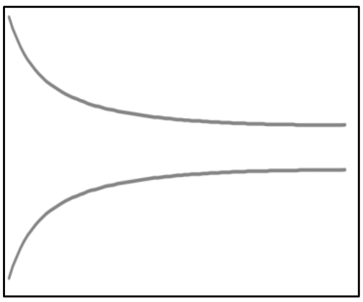

Figure 6: Convergent Nozzle Contour

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Divergent Section Design:

Region to region method of characteristics for designing the contour is employed in this section. The flow will be divided into various regions by the incident and reflected characteristics from the centre line. using θ and ν, the Mach in the regions will be estimated by employing relations between ν and θ. The nozzle is considered to be a perfect mirror at halfway So we will be working on the upper half of the nozzle only.

For sharp cornered nozzle,

$$\theta_{fan} = \nu/2$$

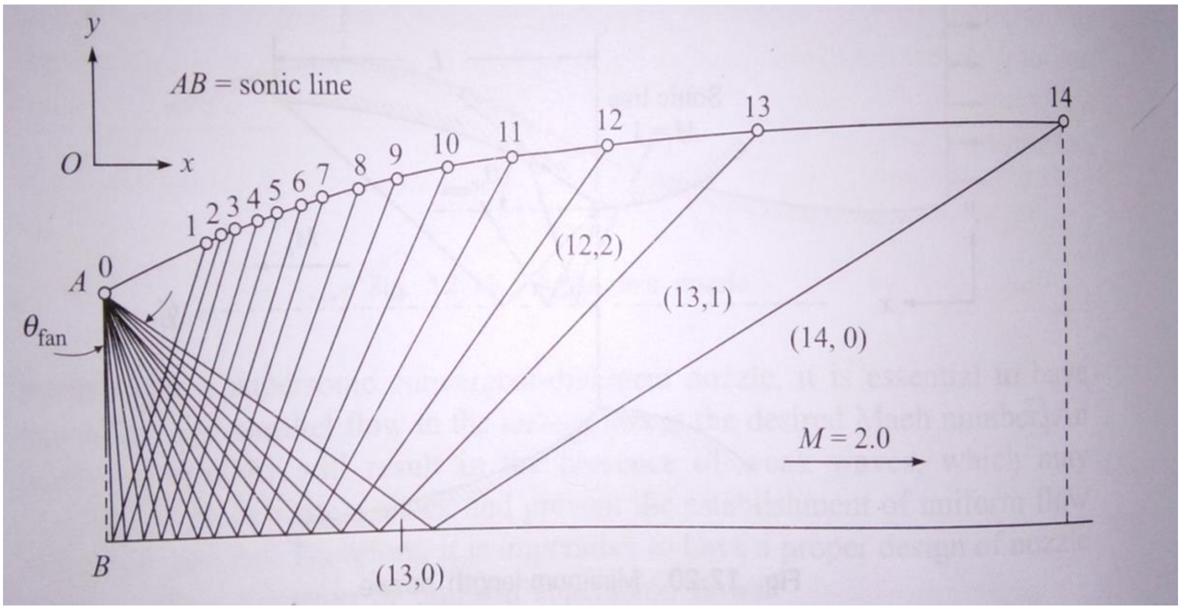

Figure 7: Divergent part Characteristics

for M = 2, by using the below ν and M relation we get the values of ν.

$$\nu\,(\text{Mach}) = \sqrt{\frac{\Upsilon+1}{\Upsilon-1}}\, tan^{-1}\sqrt{\frac{\Upsilon-1}{\Upsilon+1}}\,(Mach^2-1) - tan^{-1}\sqrt{(Mach^2-1)}$$

ν = 26.38° hence, θ fan = 13.19°

The whole expansion fan is studied in 14 sets of characteristics, the first one is a an angle of 0.19 degrees and the rest of are placed equally at a difference of 1 degrees each.. The regions are made by reflection from the centreline. For e.g., characteristic 1 makes 15 regions from (0 , 0) to (0 , 14), as shown in figure 7.

The values of θ and ν at each region can be estimated as follows.

For the regions made by the first wave,

$$\theta = 0.0°, \quad \nu = 0.0° \quad [region\ (0\,,0)]$$
$$\theta = .19°, \quad \nu = .19° \quad [region\ (0\,,1)]$$
$$\theta = 1.190°, \quad \nu = 1.190° \quad [region\ (0\,,2)]$$

For the regions made by the second wave,

$$\theta = 0.0°, \quad \nu = 0.380° \quad [region\ (1\,,0)]$$
$$\theta = 1.0°, \quad \nu = 1.380° \quad [region\ (1\,,1)]$$

Similarly, a table is constructed up to region (14, 0)

To estimate the x & y location of a contour point i, the below formula is used:

$$x_i = \frac{(A/A_t)_i - (A/A_t)_{i-1}}{2\,tan(\theta_{i-1})} y_t + x_{i-1}$$

$$y_{i=(A/A_t)_i}\ y_t$$

Where i = 1, 2, 3, … … . ,14

$\theta_{(i-1)}$ = turning angle (region $i-1$)

$(A/A_t)_i$ = area ratio at (point i)

$(A/A_t)_{i-1}$ = area ratio at (point $i-1$)

$(A/A_t)_0$ = area ratio at throat = 1

Here we are dealing with the only upper part of the Divergent section so X and Y are the lengths from the centre line of Nozzle.

The code which generates the table (x and y location) is listed in appendices. The obtained x and y points are plotted in the below figure which shows the shape of divergent part.

The Throat area is 62.53 cm^2 from our convergent part calculation and we have considered unit width, so the Y length is 31.265 cm from the centre line.

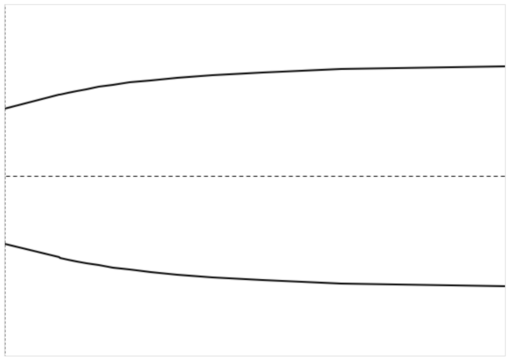

Figure 8: Divergent part upper half contour from the calculation

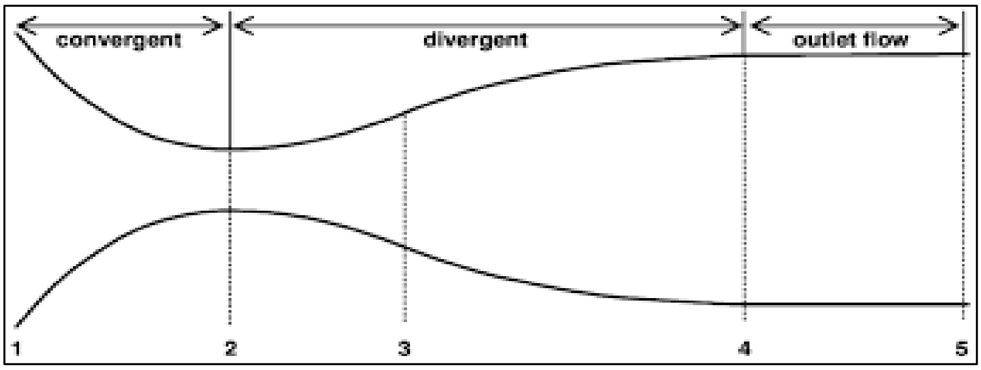

Figure 9: A typical Convergent Divergent Nozzle

III. CONCLUSION

The super-sonic nozzles are used widely across the aerospace industry. This nozzle work in a complex flow environment. Design of this kind of nozzle requires computer associated tool to achieve high accuracy. The high accuracy is required due to the fact that even a small disturbance can cause a high fluctuation in the operation of the nozzle. So, a computerised approach to iterate a large range of solution is best suited here.

The Design of convergent part is rather a simple one due to the absences of shock wave because the convergent part only deals with the subsonic flows. The length of convergent part depends on the weight and the boundary layer separation. So, there is a compromise between the weight and separation concern. The convergent part can be extremely shorter also with respect to the divergent part length. The figure below shows the convergent part as well ae divergent part and the difference between both can be seen.

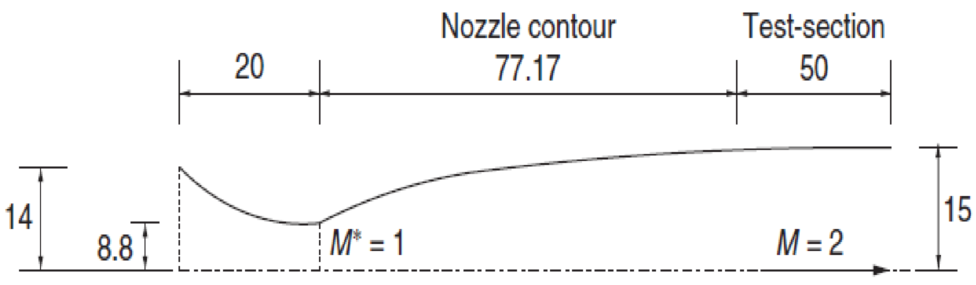

Figure 10: Supersonic Nozzle Example

The Method of Characteristics method is the best suited method for the design of the super-sonic nozzle with the least possible length. To capture the actual working condition of the nozzle one should account the viscous effect, boundary layer interaction effect etc.

In the present study the actual situation has been simplified by making various assumptions such as considering the nozzle as a 2-d nozzle. In a 3-dimensional case there are more variation along with the space and the above relation are not sufficient to study the case.

IV. FUTURE SCOPE

The present study of a super-sonic nozzle design has proved that the Method of Characteristics is a very handy and useful method for the smallest possible length nozzle design,

Future works will be focussed on 3-d nozzle design as well as on the sensitivity analysis also. The aim is also to write a code to calculate the performance of the nozzle.

Apart from that, CFD simulations will also be developed to visualize the flow pattern through the nozzle and analyse it.

At the end, a 3-D CAD model will be manufactured with help computer tools to see the real working of the nozzle.

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