

RSA ALGORITHM: A THEORETICAL STUDY AND IMPLEMENTATION

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ABSTRACT

There are many aspects in which security can be provided, even many applications too like secure payments, private communications. One such indispensable aspect is Cryptography. Though it's probably the oldest art, still the techniques used in Cryptography are a crucial means in achieving security. It not only clinches in making information restricted but also delivers various protection and security features like system security, digital signatures. So, the methods, encryption, and decryption of cryptography play a vital role in achieving the security mentioned above. The quality of the security provided will be entirely dependent on the quality of the encryption and decryption algorithms which in turn can be said to be based on the structure of mathematics and the confidentiality of key. The key can be said as the essence of encryption, as by knowing the key a person can encrypt or decrypt the information. Hence, choosing the key is the most vital process. This paper provides an overview and implementation of RSA, a public-key cryptosystem.

KEYWORDS : cryptography, public-key cryptography, RSA algorithm.

I. INTRODUCTION

The main problem that cryptography tries to solve is preventing unauthorized access to information [1]. In general, cryptography works by encrypting the plaintext into a cipher text which will be turned back into plaintext by decrypting it. Thus, the methods encryption and decryption are the crucial schemes in achieving better security. Mainly the usage of an efficient encryption algorithm helps to attain better safety to the end-user and greater affiliation for the attacker.

The RSA algorithm is a public-key cryptosystem that is widely used in digital signatures despite being published in 1977. It was first presented by R. L. Rivest, A. Shamir, and L. Adleman in the paper "A Method for Obtaining Digital Signatures and Public-Key Cryptosystems" in 1978. The essence of the RSA algorithm lies in prime factorization which is known as the unsolved problem in computer science [2] as that's what keeps RSA more secure and difficult to break. This paper implements the RSA algorithm in Java by generating very huge prime numbers.

II. THE RSA CRYPTOSYSTEM

RSA (Rivest–Shamir–Adleman) is among the first public-key cryptosystems which are widely used for secure data transmission [3]. In this Cryptosystem, the encryption key is public and is different from the decryption key which is kept as private or secret also known as a private key. This Asymmetric Encryption system uses two distinct but related keys. The Public Key, is used for encryption and the other, the Private Key, is for decryption. The Private Key is to be kept private so that only the authenticated receiver can decrypt the message. This asymmetry is based on the practical difficulty of factoring the product of two large prime numbers. A user generates and then publish a public key on the basis of two large prime numbers, along with an additional value with the key. The prime numbers that are generated must be kept as a secret. Anyone who has the public key can encrypt the message, but only the message can only be decoded by the one with the prime numbers. The security becomes much strong as it gets harder for the users to decrypt the encrypted message without the private key, as the complexity of the factorization of large integers increases. The use of two keys makes this encryption system a very complex technique. Hence, it proves to be very beneficial in terms of data security. Till date there are not any methods to defeat the system if a larger key is used for encryption.

- (A) **Public Key Cryptography** : Public Key cryptography is a method of encrypting data with two different keys i.e. private and public [4]. The public key is available for anyone to use. The private key is kept private or as a secret for each individual. The sender can encrypt a message using the receiver's public key, but that encrypted message can only be decrypted with the receiver's private key. This ensures that only the receiver will be able to read the message. Let us assume, User X wants to send a private message, 'm', to user Y. User X gets User Y's public key from mail. User X encrypts message 'm' using Y's public key. This produces a cipher text message 'c' which is sent over a mail. Upon receiving, User Y decrypts message 'c' using their private key. This results in the original message 'm'.
- (B) **RSA Cryptography** : RSA Cryptography is a cryptographic system that uses the RSA algorithm which is an asymmetric algorithm in which the two set of keys are used i.e. Public Key and Private Key. The Public Key is given to everyone and the Private key is kept private or secret [5].

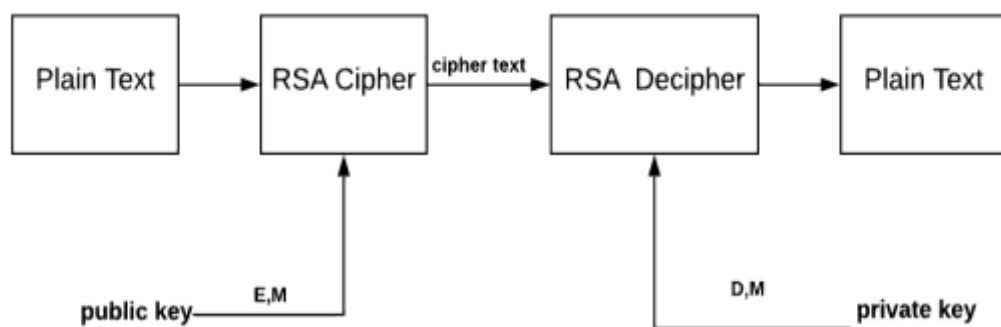


Fig-1: Working of the RSA Algorithm

(C) **The Process of RSA Algorithm :**

- The RSA algorithm involves the following steps to obtain the public and private keys: Randomly generate two primes P and Q of huge length.
- Compute the value of n which is the product of P and Q and the Euler function of n i.e. $\phi(n)$ which is $\phi(n) = (P-1) \times (Q-1)$.
- Pick an integer e which should be relatively prime with n and the $\gcd(e, n)$ should be equal to 1.
- Now the public key will be (e,n).
- Calculate the secret exponent d such that $e \cdot d \mod \phi(n) = 1$
- The private key obtained is (d,n).
- The process of Encryption i.e., encrypting the plaintext(m) to ciphertext(c) is done by: $c = m^e \mod n$.
- The process of Decryption involves taking the ciphertext(c) obtained and generating the original plaintext(m): $m = c^d \mod n$

(D) **Security and Applications :**

The security of the RSA algorithm depends on the integer factorization method. Thus, by taking two prime numbers of large lengths, it takes a very long time to perform factorization thereby making it more secure. In general, prime factorization is considered to be an unsolvable problem in computer science as it takes a very long time even for the supercomputers; hence it's vital to generate P and Q values as primes and of very huge length. But ultimately decomposing the n value is the main way to attack the algorithm.

The HTTPS (Hypertext Transfer Protocol Secure) uses a secure communication protocol called TLS (Transport Layer Security) which was previously the SSL (Secure Sockets Layer) [6]. This SSL helps to encrypt the data transfer between a client and a server and by using the RSA encryption it achieves this. In general, the data transmission over the SSL can be classified into two steps:

- SSL handshake
- The actual transfer of data

The SSL handshake is where the communication begins over the SSL. It's where the RSA algorithm is used to generate public key and establish a secure connection. For example, basically a server generates two prime numbers and multiplies them and generates a public key(e). Now any client that wants to communicate i.e., transfer the data to the server takes that public key(e), encrypts the data, and sends it. Now the server as it knows the private key it decrypts the data using it. After this initial handshake the SSL uses the AES algorithm i.e a symmetric encryption algorithm for the actual transfer of data. In general, the SSL uses 128 or 256 digits for the keys, and in order to break these keys, a very vast amount of computing power is required [7]. Thus, RSA is said to be very hard to break and again it depends mainly on the length of the prime numbers generated.

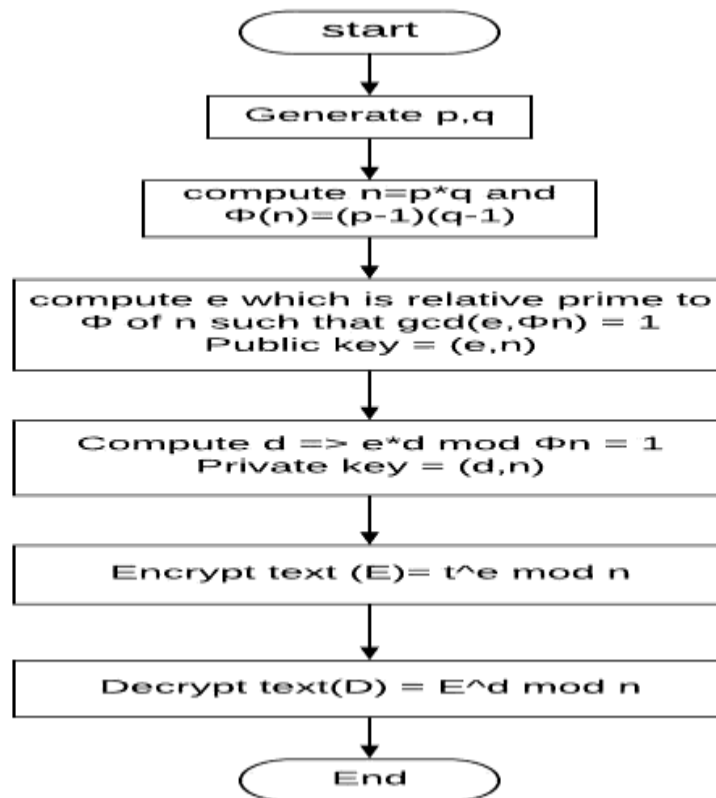


Fig-2: Flowchart Describing the Process of the RSA Algorithm

III. IMPLEMENTATION

A. Code

/* You can find the code in my GitHub

Repo

itsknk/CryptographySuite */

```
public class
```

```
RSAalgo {
```

```
    BigInteger p;
```

```
    BigInteger q;
```

```
    BigInteger n;
```

```
    BigInteger z;
```

```
    BigInteger e;
```

```
BigInteger d;
int bitlength = 1024
; Random r;
public RSAalgo
(){ r = new Random();
//generates a big prime number within the specified length
p = BigInteger . probablePrime(bitlength, r);
q = BigInteger . probablePrime(bitlength, r);
System . out . println( "The Value of p: " + p);
System . out . println( "The Value of q: " + q);
//Calculating N i.e, N=p*q
n = p . multiply(q);
//Calculating φN i.e, φN=(p-1)(q-1)
z = p . subtract(BigInteger . ONE ) . multiply(q . subtract(BigInteger . ONE ));
System . out . println( "The Value of N is: " + n);
System . out . println( "The Value of φN is: " + z);
//generates e which will be a relative prime to φN
e = BigInteger . probablePrime(bitlength / 2 , r);
//to check if 1<e<φ, such that gcd(e,φ)=1
while (z . gcd(e) . compareTo(BigInteger . ONE )>0&&e . compareTo(z) < 0 ) {
e . add(BigInteger . ONE );
}
//calculating d i.e, d should be = e*dmod(φN)=1 d = e . modInverse(z);
//Public Key = (e,N)
System . out . println( "Public Key generated: ("
+ e + "," + n + ")" );
//Private Key = (d,N)
System . out . println( "Private Key generated: ("
+ d + "," + n + ")" );
}
public RSAalgo (BigInteger e , BigInteger d , BigInteger n )
{ this . e = e;
this . d = d; this . n = n;
}
public static void main (String [] args ) throws IOException
{ RSAalgo rsa = new RSAalgo();
DataInputStream inp =new DataInputStream(System . in); String question ;
System.out.println( "Enter the plain text:" );
```

```

question = inp.readLine();
System.out.println( "Encrypting String: " + question);

//converting the string to Bytes
System.out.println( "String in Bytes: " + bytesToString(question . getBytes()));

// encrypt
byte [] encrypted = rsa.encrypt(question . GetBytes());
System.out.println( "Encrypted String in Bytes: " + bytesToString(encrypted));

// decrypt
byte [] decrypted = rsa.decrypt(encrypted);
System.out.println( "Decrypted String in Bytes: " + bytesToString(decrypted));
System.out.println( "Decrypted String: " + new String(decrypted));
}

public static String bytesToString ( byte [] encrypted )
{ String something = "" ;

for ( byte b : encrypted) { something += Byte . toString(b);

}

return something;

}

//Encryption

public byte [] encrypt ( byte [] msg ){ return ( new

BigInteger(msg) . modPow(e,n) . toByteArray()); //to encrypt, msge*mod(N)

}

//Decryption

public byte [] decrypt ( byte [] msg )
{ return ( new

BigInteger(msg) . modPow(d,n) . toByteArray()); //to decrypt, msg^d*mod(N)

}
}

```

B. Output

```

C:\Users\prajna> java H3a3iq
The Value of p: 13348894715648468887000111767456183817916672278252893528156744305789204412129621736169966159178794564388790864881535515762634888485676
982255922238737636260295125209336679958361133624879658166789792855672378432866285133081612665876348369409549283+3933888997988568989276813814784984600
3599711805978852123438323
The Value of q: 142074594311578726345667163824411641586568446595385327996124591466594588393358136551410924110453557135742320339567249114844253570628
4928669585241858729851664349090298849174071786714895323613744802829829596671224348570045874334558417991330594948392720956956993192823186896858430055
3624590386240441127792963

```

Fig.3 : Huge Prime Numbers are Generated for p and q

```

The Value of N is: 1906532118952306696603477875648933520489388496548248539769333538997405500064566396241549988896447857536788298891441764834389271891
8333927454618227202511729582971586466666180868362491558366685887269715164253469910433135124842960763639363011046977359703442104079135838288275884783792
26693816819892836247690366744583165831215975322250080409697821354828745879625544728682692476363488466824863669723571746439792777647809372631162681422648
880233867367263712558613009991597214484034327782927053731493943771990659138573889758471065104012788728666040413283848806744825482541038423389337831751
827414766815956568864572671258899649
The Value of phi is: 1906532118952306696603477875648933520489388496548248539769333538997405500064566396241549988896447857536788298891441764834389271891
833392745461822720251172958297158646666618086836249155836668588726971516425346991043313512484296076363936301104697735970344210407913583828827588478379
8276938168198928362476903667169578118653622119018157948278848751436787117273772126021585781596424768642716387867364026152481364073273978775157377327
869761316431325248371689378298039259888124272343417017838833671186731441648811937401447587991382711832287611178231243062069657468717553483848096274498
1581781448166736258472353378907586188

```


Fig.4 : n and ϕn are Calculated

[illegible]

Fig.5 : The Public(e,n) and Private(d,n) Keys Obtained

```

Encrypting String: one hundred years of solitude
String in Bytes: 11111010112104171101001140110031211010711411532111023213511100105101117100101
Encrypted String in Bytes: 6100-3347-35-54-46-61106451007058-51125107-77-110-6250500000913082-102-724305-0611120337-7034-115-100-3043-22121-1322-9-975
121-346-30-1210-461254100112-14055141-42110415-00-53-61-0790618623-107-91-822114-06-8373-111741-35-96-1719595431611214-130-66-52-108-26-1069951-14486
105-1121158323-52-75660512083-17-9710095-127-3012-119-70-809919-29-341142126-6012-0410314-2377-97-5497-1938001-690-10007493527-005091-54-72-3529131
1995-117-6999-94-06-1260-40105-1918007-12165-10-115-65-26-24121041089999326-83-102-1211262212464-88-2227818389-11328-125-26-1122-71-06117112-57-30
105-3-08-36790448-11-6758626929330733-53-125-5101-77-10016179400069-11-27-5412206-11199-131-79-61
Decrypted String in Bytes: 111110101321041711010011410110031211019711411532111023213511100105101117100101
Decrypted String: one hundred years of solitude

```

Fig.6 : Encryption and Decryption of the text “one hundred years of solitude”

C. Explanation

The implementation of RSA is somewhat complex as the whole concept of making RSA secure lies on the factorization of two large integers [8], the primes generated should be very huge in length. Thus, instead of using integer variables, we use Big Integer for p and q to generate random huge prime numbers. As mentioned above in the process we find the values of n , ϕn first. Then we choose an integer e which will be relatively prime to ϕn and $\gcd(e, \phi n) = 1$. The public

key(e,n) and private key(d,n) will be generated as per the process mentioned above. In Java, we convert the data entered i.e., the plaintext into bytes and we use these bytes to encrypt the plain text, similarly, we use the encrypted text and decrypt it and convert the decrypted bytes into the string which will return the entered plain text.

IV. CONCLUSION

This paper explored the core concepts of cryptography i.e., encryption and decryption, then public-key cryptography mainly the RSA cryptosystem. The encryption and decryption algorithms play a vital role in achieving security and the quality of those algorithms shows the quality of the security provided. In general, in a public-key cryptosystem, the encryption key will be public and will be different from the secret key which will be used for decryption. The RSA is one such public-key cryptosystem or the asymmetric cryptosystem which is widely used. The core idea of attaining security in RSA is by factorization of integers and hence in RSA, we use two very large prime numbers as their factorization is very difficult and considered as an unsolvable problem in computer science. This paper provided the implementation of the RSA algorithm in Java and this code can be utilized to implement the RSA in the required applications. The data generated to or from the server over the HTTPS is encrypted and the SSL certificates of the websites i.e. where HTTPS is applied depends on the RSA. But the future scope involves the rise in the usage of other cryptosystems like Diffie-Hellmann and Elliptic curve instead of RSA, as the recent research has exhibited that the RSA keys of length 2048 bits can be effectively demoted by padding oracle attack and other [9].

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