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CREEPING MICROPOLAR FLOW PAST A POROUS APPROXIMATE SPHERE

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ABSTRACT

A drag formular for the creeping flow of an incompressible micropolar fluid past a porous approximate sphere with permeability, k, assuming uniform stream velocity far away from the body and along its axis of symmetry, is proposed and explored with varying values of the permeability parameter η and the coupling number N. Expressions for the principle inner and outer flow regions are determined by matching the solutions of Stokes' equations with Brinkman equations for the inner region. The micropolar parameter m is studied numerically. These results involving a porous sphere, approximate sphere and impermeable sphere, as well as the classical Newtonian flows are obtained as special cases.

Keywords: Micropolar Fluid, Non-Newtonian, Porous, Approximate Sphere.

I. INTRODUCTION

Fluids which enjoy their own complex micro-structures cannot be accurately modelled by the traditional approach to fluid dynamics, as additional balance laws and constitutive relations are needed to describe such complex behaviors. This prompted the emergence of several micro-fluidic theories as early as the 1920's.

Leading British mathematical physicist George Barker Jeffery investigated the micro-structural effects in fluids by studying the motion of ellipsoidal particles immersed in a viscous fluid and found that the value of fluid viscosity increased due to the presence of these particles (Jeffery, 1922). American mathematician J.L. Ericksen is accredited with developing field equations that take into account the micro-structure of the fluid (Ericksen, 1960). Navy researchers J.W. Hoyt and A.G. Fabula were amongst the first to make significant contributions in the study of drag inducing additives for military applications (Hoyt and Fabula, 1964). W.M. Vogel and A.M. Patterson were instrumental in noting that polymer additives near a rigid body have a lower skin friction than one without additives (Vogel and Patterson, 1964). This is significant as the concept of drag reduction in external flow circumstances can be effectually used for reducing the drag on full-scale ships as well as other submerged vessels.

In 1964 at the Naval Hydrodynamic conference in Bergen-Norway, Turkish-American engineering scientist, Professor Ahmed C. Eringen of Princeton University, presented his Microfluid theory, (Eringen, 1964). Eringen extended the theory of micro-continua to a special case of non-Newtonian fluids which he named 'simple microfluids' where the deformable fluid micro-elements exhibit local motions and inertia. With this generalisation of the classic Navier-Stokes equations, Eringen opened the door to modelling more complex fluids that are useful in engineering problems. Neglecting the deformation, a sub-class of his "Simple Microfluids" called Micropolar fluids was then introduced by Eringen (Eringen, 1966). While simple microfluids are modelled through 19 partial differential equations, Micropolar fluids only require 6, as they are physically represented as possessing rigid elements with micro-rotational effects and inertia. Significant aapplications of Micropolar fluid theory can be found in lubrication theory, with the lubricating fluids within bearings, the physics of liquid crystals and drilling fluids of the oil industry.

The problem under current observation extends the research done in 1995 by T.K.V. Iyengar and D. Srinivasacharya who published 'Stokes Flow of an Incompressible Micropolar Fluid past an Approximate Sphere' (Srinivasacharya, 1995) and Professor H. Ramkissoon's work in flow past spheres, through porous spheres and spheroid mediums (Ramkissoon, 1975, 1997). This paper presents the problem of 'Creeping Micropolar Flow Past a Porous Approximate Sphere'.

II. STATEMENT OF THE PROBLEM

Due to the geometry of the problem spherical polar coordinates, (r, θ, φ) are used. Let us consider the steady flow of an incompressible micropolar fluid moving past a porous approximate sphere. Assume that uniform velocity, U, exists far away from the body and along the axis of symmetry $\theta = 0$, which corresponds to the z-axis. Fluid flowing outside of the porous approximate sphere is Stokesian in nature while flow within the porous object is governed by the Darcy-Brinkman model for porous media flows, (Brinkman, 1947). The *external* micropolar field equations are under Stokesian assumption in the absence of body force and couples, (Eringen, 1966), are:

$$\nabla \cdot \vec{q}^{(1)} = 0 \tag{1}$$

$$-\nabla p^{(1)} + \kappa \nabla \times \vec{\omega}^{(1)} - (\mu+\kappa)\nabla \times \nabla \times \vec{q}^{(1)} = 0 \tag{2}$$

$$-2\kappa\vec{\omega}^{(1)} + \kappa\nabla \times \vec{q}^{(1)} - \gamma\nabla \times \nabla \times \vec{\omega}^{(1)} + (\alpha+\beta+\gamma)\nabla\left(\nabla \cdot \vec{\omega}^{(1)}\right) = 0 \tag{3}$$

The *internal* micropolar field equations are based upon the Brinkman model for flow within a porous medium, (Eringen, 1966), (Brinkman, 1947), they are given by:

$$\nabla \cdot \vec{q}^{(2)} = 0 \tag{4}$$

$$\frac{\mu}{\kappa}\vec{q}^{(2)} + \nabla p^{(2)} - \kappa\nabla \times \vec{\omega}^{(2)} + (\mu+\kappa)\nabla \times \nabla \times \vec{q}^{(2)} = 0 \tag{5}$$

$$-2\kappa\vec{\omega}^{(2)} + \kappa\nabla \times \vec{q}^{(2)} - \gamma\nabla \times \nabla \times \vec{\omega}^{(2)} + (\alpha+\beta+\gamma)\nabla\left(\nabla \cdot \vec{\omega}^{(2)}\right) = 0 \tag{6}$$

$\vec{q}$, $\vec{\omega}$ and p denote respectively the velocity vector, microrotation vector and the fluid pressure with super-scripts $i = 1, 2$ signifying the respective external and internal flows. k represents the permeability of the porous medium and the material constants μ, κ, α, β and γ satisfy the usual inequalities, (Eringen, 1966):

$$2\mu+\kappa \geq 0, \qquad \kappa \geq 0, \qquad 3\alpha+\beta+\gamma \geq 0, \qquad \gamma \geq |\beta|$$

Consider the equation of an approximate sphere, (Srinivasacharya, 2003):

$$r = a\left[1 + \sum_{\tilde{m}=2}^{\infty} \beta_{\tilde{m}} V_{\tilde{m}}(\zeta)\right] \tag{7}$$

$$\zeta = \cos\theta$$

$$V_{\tilde{m}}(\zeta) = \left[\frac{P_{\tilde{m}-2}(\zeta) - P_{\tilde{m}}(\zeta)}{2\tilde{m}-1}\right] \tag{8}$$

$P_{\tilde{m}}(\zeta)$ and $V_{\tilde{m}}(\zeta)$ are respectively called the Legendre and Gegenbauer functions of the first kind and of order m. Due to simplicity considerations, coefficient $\beta_{\tilde{m}}$ is assumed adequately small, so that its squares and higher powers are disregarded.

As the flow is axisymmetric about the z-axis, all the flow functions are represented independent of φ and the velocity and microrotation vectors are chosen as follows:

$$\vec{q}^{(i)} = u^{(i)}(r,\theta)\vec{e}_r + v^{(i)}(r,\theta)\vec{e}_\theta \tag{9}$$

$$\vec{\omega}^{(i)} = v_\varphi^{(i)}(r,\theta)\vec{e}_\varphi \tag{10}$$

The Stokes stream function, $\psi(r,\theta)$, is introduced:

$$u^{(i)} = -\frac{1}{r^2 \sin\theta}\frac{\partial\psi^{(i)}}{\partial\theta}, \qquad v^{(i)} = \frac{1}{r\sin\theta}\frac{\partial\psi^{(i)}}{\partial r} \tag{11}$$

and the following dimensionless variables are utilised,

$$r = a\tilde{r}, \qquad \psi^{(i)} = Ua^2\psi^{(i)}, \qquad p^{(i)} = \frac{\mu U}{a}\tilde{p}^{(i)}, \qquad v_\varphi^{(i)} = \frac{U}{a}\tilde{v}_\varphi^{(i)} \tag{12}$$

Equations (11) and (12) are substituted into (1) - (6) to give,

$$-\frac{\partial p^{(1)}(r,\theta)}{\partial r} - \left(\frac{1}{1-N}\right)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(E^2\psi^{(1)}(r,\theta)\right)\left(\frac{N}{1-N}\right)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(r\sin\theta\, v_\varphi^{(1)}(r,\theta)\right) = 0 \tag{13}$$

$$-\frac{1}{r}\frac{\partial p^{(1)}(r,\theta)}{\partial\theta}+\left(\frac{1}{1-N}\right)\frac{1}{r\sin\theta}\frac{\partial}{\partial r}\left(E^2\psi^{(1)}(r,\theta)\right)-\frac{N}{1-N}\frac{1}{r\sin\theta}\frac{\partial}{\partial r}\left(r\sin\theta\, v_\varphi^{(1)}(r,\theta)\right)=0 \quad (14)$$

$$-2v_\varphi^{(1)}(r,\theta)+\frac{1}{r\sin\theta}\left(E^2\psi^{(1)}(r,\theta)\right)+\frac{2-N}{m^2}\left[\nabla^2-\frac{1}{r^2\sin^2\theta}\right]v_\varphi^{(1)}(r,\theta)=0 \quad (15)$$

$$-\frac{\partial p^{(2)}(r,\theta)}{\partial r}-\left(\frac{1}{1-N}\right)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(E^2\psi^{(2)}(r,\theta)\right)+\left(\frac{N}{1-N}\right)\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(r\sin\theta\, v_\varphi^{(2)}(r,\theta)\right)+\eta^2\frac{1}{r^2\sin\theta}\frac{\partial\psi^{(2)}}{\partial\theta}=0 \quad (16)$$

$$-\frac{1}{r}\frac{\partial p^{(2)}(r,\theta)}{\partial\theta}+\left(\frac{1}{1-N}\right)\frac{1}{r\sin\theta}\frac{\partial}{\partial r}\left(E^2\psi^{(2)}(r,\theta)\right)-\left(\frac{N}{1-N}\right)\frac{1}{r\sin\theta}\frac{\partial}{\partial r}\left(r\sin\theta\, v_\varphi^{(2)}(r,\theta)\right)-\eta^2\frac{1}{r\ \sin\theta}\frac{\partial\psi^{(2)}}{\partial r}=0 \quad (17)$$

$$-2v_\varphi^{(2)}(r,\theta)+\frac{1}{r\sin\theta}\left(E^2\psi^{(2)}(r,\theta)\right)+\frac{2-N}{m^2}\left[\nabla^2-\frac{1}{r^2\sin^2\theta}\right]v_\varphi^{(2)}(r,\theta)=0 \quad (18)$$

where,

$$E^2=\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}-\frac{\cot\theta}{r^2}\frac{\partial}{\partial\theta}\right),$$

$$\nabla^2=\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}\right)$$

$$\nabla^2=\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}\right)$$

$$\eta^2=\frac{a^2}{k},\qquad N=\frac{\kappa}{(\mu+\kappa)}\ \text{and}\ m^2=\frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}a^2 \quad (19)$$

E^2 and ∇^2 are the Stokesian and Laplacian operators respectively, k refers to the permeability of the porous medium, N is the coupling number and m is the micropolar parameter.

Eliminating pressure from (13), (14), (16) and (17) gives,

$$E^4\left(E^2-m^2\right)\psi^{(1)}(r,\theta)=0 \quad (20)$$

$$E^2\left(E^2-\alpha^2\right)\left(E^2-\beta^2\right)\psi^{(2)}(r,\theta)=0 \quad (21)$$

where,

$$(\alpha^2+\beta^2)=\eta^2(1-N)+m^2\ \text{and}\ \alpha^2\beta^2=\frac{2(1-N)}{2-N}\eta^2 m^2 \quad (22)$$

Substituting (20) and (21) into the remaining governing equations, (15) and (18), yields general expressions for internal and external microrotation vectors, $v_\varphi^{(i)}(r,\theta)$.

$$v_\varphi^{(1)}(r,\theta)=\frac{1}{2r\sin\theta}\left[E^2\psi^{(1)}(r,\theta)+\frac{2-N}{Nm^2}E^4\psi^{(1)}(r,\theta)\right] \quad (23)$$

$$v_\varphi^{(2)}(r,\theta) = \frac{1}{2r\sin\theta}\left[E^2\psi^{(2)}(r,\theta) + \left(\frac{2-N}{Nm^2}\right)\left[E^4\psi^{(2)}(r,\theta) - \eta^2(1-N)E^2\psi^{(2)}(r,\theta)\right]\right] \tag{24}$$

III. BOUNDARY CONDITIONS

Dimensional boundary conditions on a rigid surface are: continuity of the velocity, pressure and tangential stress components, the hyperstick condition on the microrotation vector, the usual impositions of non-singularity within the flow field and uniform flow far away from the body.

In terms of the Stokes stream function these boundary conditions are:

$$\psi^{(1)}(r,\theta) = \psi^{(2)}(r,\theta), \qquad p^{(1)}(r,\theta) = p^{(2)}(r,\theta),$$

$$\psi_r^{(1)}(r,\theta) = \psi_r^{(2)}(r,\theta). \qquad v_\varphi^{(1)}(r,\theta) = 0, \tag{25}$$

$$\psi_{rr}^{(1)}(r,\theta) = \psi_{rr}^{(2)}(r,\theta). \qquad v_\varphi^{(2)}(r,\theta) = 0$$

$$\psi^{(1)}(r,\theta) \to r^2\sin^2\theta \quad as \quad r\to\infty$$

$$\psi^{(2)}(r,\theta) \text{ is finite at } r = 0 \tag{26}$$

IV. SOLUTION OF THE PROBLEM

On the exterior of the porous approximate sphere, the solutions of the stream function, microrotation vector and pressure are found to be,

$$\psi^{(1)}(r,\theta) = r^2 V_2(\zeta) + \sum_{n=2}^{\infty}\left[B_n^{(1)} r^{-n+1} + D_n^{(1)} r^{-n+3} + E_n^{(1)}\sqrt{r}K_{n-\frac{1}{2}}(mr)\right]V_n(\zeta) \tag{27}$$

$$v_\varphi^{(1)}(r,\theta) = \frac{1}{r\sin\theta}\left\{\begin{array}{c}\left[\left(\frac{m^2}{N}\right)E_2^{(1)}\sqrt{r}\,K_{\frac{3}{2}}(mr) - D_2^{(1)}r^{-1}\right]V_2(\zeta) + \\ \sum_{n=3}^{\infty}\left[D_n^{(1)}\left(\frac{6-4n}{2}\right)r^{-n+1} + \left(\frac{m^2}{N}\right)E_n^{(1)}\sqrt{r}\,K_{n-\frac{1}{2}}(mr)\right]V_n(\zeta)\end{array}\right\} \tag{28}$$

$$p^{(1)}(r,\theta) = \frac{2-N}{2(1-N)}\left[D_2^{(1)}r^{-2}P_1(\zeta) + \sum_{n=3}^{\infty}D_n^{(1)}\left(\frac{6-4n}{n}\right)r^{-n}P_{n-1}(\zeta)\right] \tag{29}$$

On the interior of the porous approximate sphere, the solutions of the stream function, microrotation and pressure terms are found to be,

$$\begin{aligned}\psi^{(2)}(r,\theta) = {} & \left[A_2^{(2)}r^2 + D_2^{(2)}\sqrt{r}\,I_{\frac{3}{2}}(\alpha r) + F_2^{(2)}\sqrt{r}\,I_{\frac{3}{2}}(\beta r)\right]V_2(\zeta) + \\ & \sum_{n=2}^{\infty}\left[A_n^{(2)}r^n + D_n^{(2)}\sqrt{r}\,I_{n-\frac{1}{2}}(\alpha r) + F_n^{(2)}\sqrt{r}\,I_{n-\frac{1}{2}}(\beta r)\right]V_n(\zeta)\end{aligned} \tag{30}$$

$$v_\varphi^{(2)}(r,\theta) = \frac{1}{r\sin\theta}\left\{\begin{array}{c}\left[D_2^{(2)}A_\alpha\sqrt{r}\,I_{\frac{3}{2}}(\alpha r) + F_2^{(2)}A_\beta\sqrt{r}\,I_{\frac{3}{2}}(\beta r)\right]V_2(\zeta) + \\ \sum_{n=3}^{\infty}\left[D_n^{(2)}A_\alpha\sqrt{r}\,I_{n-\frac{1}{2}}(\alpha r) + F_n^{(2)}A_\beta\sqrt{r}\,I_{n-\frac{1}{2}}(\beta r)\right]V_n(\zeta)\end{array}\right\} \tag{31}$$

$$p^{(2)}(r,\theta) = \frac{2-N}{1-N}\left\{\left[\frac{\alpha^2\beta^2}{2m^2}A_2^{(2)}r\right]P_1(\zeta) + \frac{\alpha^2\beta^2}{2m^2}\sum_{n=3}^{\infty}\left[\frac{1}{(n-1)}A_n^{(2)}r^{n-1}\right]P_{n-1}(\zeta)\right\} \tag{32}$$

where,

$$A_{\alpha} = \frac{\alpha^2 \{ [N\, m^2 - (2 - N)\,(1 - N)\,\eta^2] + (2 - N)\alpha^2\}}{2\, N\, m^2}$$

$$A_{\beta} = \frac{\beta^2 \{ [N\, m^2 - (2 - N)\,(1 - N)\,\eta^2] + (2 - N)\,\beta^2\}}{2\, N\, m^2} \quad (33)$$

V. DETERMINATION OF ARBITRARY CONSTANTS

Due to simplicity considerations, the surface given by (7) is considered in the following form:

$$\sigma^k = \left(\frac{r}{a}\right)^k \cong 1 + k\, \beta_{\bar{m}}\, V_{\bar{m}}(\zeta) \quad (34)$$

Upon comparison of the stream function and microrotation expressions in the flow region outside the porous approximate sphere, with the ones obtained for flow of a micropolar liquid past a porous sphere, (Srinivasacharya, 2004), it is observed that the terms involving B_n, D_n, E_n for $n > 2$ are the terms absent in (Srinivasacharya, 2004).

The body under consideration is an approximate porous sphere and the motion is not much different to that which occurs when the surface is a perfect porous sphere. All coefficients of B_n, D_n, E_n for $n > 2$ will be of order $\beta_{\bar{m}}$. Henceforth, in these expressions involving B_n, D_n, E_n, there is disregard shown for the departure from a perfectly spherical form and as such r is set at 1 whilst implementing the boundary conditions. This indicates that within the previously stated allowances of B_n, D_n, E_n that $\sigma^k = 1$. The boundary conditions can now be restated in terms of σ where $\sigma^k = 1$.

For simplicity, the stream functions, microrotation vectors and pressure terms were each set to a general form, λ, illustrated below:

$$\lambda^{(i)}(r, \theta) = \lambda_A^{(i)}(r, \theta) + \sum_{n=3}^{\infty} \lambda_B^{(i)}(r, \theta) \quad (35)$$

Using the above technique, the boundary conditions previously presented are re-expressed below:

$$\psi_A^{(1)}(\sigma, \theta) - \psi_A^{(2)}(\sigma, \theta) + \sum_{n=3}^{\infty} \left(\psi_B^{(1)}(\sigma, \theta) - \psi_B^{(2)}(\sigma, \theta)\right) = 0 \quad (36)$$

$$\psi_{Ar}^{(1)}(\sigma, \theta) - \psi_{Ar}^{(2)}(\sigma, \theta) + \sum_{n=3}^{\infty} \left(\psi_{Br}^{(1)}(\sigma, \theta) - \psi_{Br}^{(2)}(\sigma, \theta)\right) = 0 \quad (37)$$

$$\psi_{Arr}^{(1)}(\sigma, \theta) - \psi_{Arr}^{(2)}(\sigma, \theta) + \sum_{n=3}^{\infty} \left(\psi_{Brr}^{(1)}(\sigma, \theta) - \psi_{Brr}^{(2)}(\sigma, \theta)\right) = 0 \quad (38)$$

$$p_A^{(1)}(\sigma, \theta) - p_A^{(2)}(\sigma, \theta) + \sum_{n=3}^{\infty} \left(p_B^{(1)}(\sigma, \theta) - p_B^{(2)}(\sigma, \theta)\right) = 0 \quad (39)$$

$$\nu_{\varphi A}^{(1)}(\sigma, \theta) + \sum_{n=3}^{\infty} \nu_{\varphi B}^{(1)}(\sigma, \theta) = 0 \quad (40)$$

$$\nu_{\varphi A}^{(2)}(\sigma, \theta) + \sum_{n=3}^{\infty} \nu_{\varphi B}^{(2)}(\sigma, \theta) = 0 \quad (41)$$

Substitutions of equations (27) to (33) into (36) to (41) resulted in the solution of the following constants:

$$D_2^{(1)} = \frac{-3\,\alpha^2\,\beta^2\,I_{\frac{3}{2}}(\alpha)\,K_{\frac{3}{2}}(m)\left(\alpha^{\,2}\,A_\beta\ -\ \beta^2\,A_\alpha\right)I_{\frac{3}{2}}(\beta)\,m}{D_\Lambda}$$

$$E_2^{(1)} = \frac{-3\,N\,\alpha^2\,\beta^2\left(\alpha^2\,A_\beta\ -\ \beta^2\,A_\alpha\right)I_{\frac{3}{2}}(\alpha)\,I_{\frac{3}{2}}(\beta)}{D_\Lambda\,m}$$

$$B_2^{(1)} = \frac{1}{D_\Lambda.m}\left\{\begin{matrix} -\alpha^2\,\beta^2\left\{\left(\alpha^2 A_\beta - \beta^2 A_\alpha\right)\left[m^2 K_{\frac{3}{2}}(m) + N\left(mK_{\frac{1}{2}}(m) + 3K_{\frac{3}{2}}(m)\right)\right] I_{\frac{3}{2}}(\alpha)\,I_{\frac{3}{2}}(\beta) - \right. \\ (2-N)\left[\begin{matrix}(A_\alpha - A_\beta)3\,m^2\,K_{\frac{3}{2}}(m)\,I_{\frac{3}{2}}(\alpha)\,I_{\frac{3}{2}}(\beta) + \\ m^2 K_{\frac{3}{2}}(m)\left(\alpha\,A_\beta\,I_{\frac{3}{2}}(\beta)\,I_{\frac{1}{2}}(\alpha) - \beta\,A_\alpha I_{\frac{3}{2}}(\alpha)\,I_{\frac{1}{2}}(\beta)\right)\end{matrix}\right]\end{matrix}\right\}$$

$$A_2^{(2)} = \frac{3\,m^3\,I_{\frac{3}{2}}(\alpha)\,I_{\frac{3}{2}}(\beta)\left(\alpha^2 A_\beta - \beta^2 A_\alpha\right)K_{\frac{3}{2}}(m)}{D_\Lambda}, \qquad D_2^{(2)} = \frac{3\,\alpha^2\,\beta^2\,m\,(2-N)\,A_\beta\,K_{\frac{3}{2}}(m)\,I_{\frac{3}{2}}(\beta)}{D_\Lambda}$$

$$F_2^{(2)} = \frac{-3\,\alpha^2\,\beta^2\,m\,(2-N)\,A_\beta\,K_{\frac{3}{2}}(m)\,I_{\frac{3}{2}}(\alpha)}{D_\Lambda}$$

where:

$$D_\Lambda = \left[(3\,m^3 + \alpha^2\,\beta^2)\,K_{\frac{3}{2}}(m) - \alpha^2\,\beta^2 N\,K_{\frac{1}{2}}(m)\right]\left(\alpha^2 A_\beta - \beta^2 A_\alpha\right) I_{\frac{3}{2}}(\alpha)\,I_{\frac{3}{2}}(\beta) + \left[\alpha\,A_\beta\,I_{\frac{3}{2}}(\beta)\,I_{\frac{1}{2}}(\alpha) - \beta\,A_\alpha\,I_{\frac{3}{2}}(\alpha)\,I_{\frac{1}{2}}(\beta)\right](2-N)\,m\,\alpha^2\beta^2\,K_{\frac{3}{2}}(m) \quad (42)$$

VI. DETERMINATION OF DRAG

In 1976, H. Ramkissoon and S. R. Majumdar presented their formulae for drag on the sphere, (Ramkissoon, 1976):

$$D = 4\,\pi\,(2\,\mu + K)\lim_{r\to\infty}\frac{r\,(\psi\ -\ \psi_\infty)}{R^2} \quad (43)$$

where ψ_∞ is the stream function corresponding to the motion of a micropolar fluid creeping past an axisymmetric body far off at infinity.

As a new result, a revised expression for drag force for a porous approximate sphere is given by,

$$D_{G_{Micropolar}} = 4\,\pi\,(2\,\mu + K)\left[\frac{D_2^{(1)}}{2} + D_{\tilde{m}-2}^{(1)}\,\frac{V_{\tilde{m}-2}(\zeta)}{sin^2\,\theta} + D_{\tilde{m}}^{(1)}\,\frac{V_{\tilde{m}}(\zeta)}{sin^2\,\theta} + D_{\tilde{m}+2}^{(1)}\,\frac{V_{\tilde{m}+2}(\zeta)}{sin^2\,\theta}\right]$$

Or in a more compact form,

$$D_{G_{Micropolar}} = 4\,\pi\,(2\,\mu + K)\left[\frac{D_2^{(1)}}{2} + D_n^{(1)}\,\frac{V_n(\zeta)}{sin^2\,\theta}\right]_{n\,=\,\tilde{m}-2,\ \tilde{m},\ \tilde{m}+2} \quad (44)$$

Using the boundary conditions (36) - (41), a linear system of equations in the arbitrary coefficients $B_2^{(1)}, D_2^{(1)}, E_2^{(1)}, A_2^{(2)}, D_2^{(2)}, F_2^{(2)}$ and $A_n^{(2)}, B_n^{(1)}, D_n^{(1)}, D_n^{(2)}, E_2^{(1)}, F_n^{(2)}$ was found. This system of equations was solved and the expressions for $D_2^{(1)}$and $D_n^{(1)}$
were obtained as:

$$D_2^{(1)} = \frac{3\, m\, \alpha^2\, \beta^2\, K_{\frac{3}{2}}(m)\ I_{\frac{3}{2}}(\alpha)\, I_{\frac{3}{2}}(\beta) A_\gamma}{-A_\gamma I_{\frac{3}{2}}(\alpha) I_{\frac{3}{2}}(\beta)\left[3m^{\,3}\, K_{\frac{3}{2}}(m)\ +\ \alpha^2\,\beta^2\ \left(2m\, K_{\frac{3}{2}}(m)\ -\ N\, K_{\frac{1}{2}}(m)\right)\right] + (2-N)m\alpha^2\beta^2 K_{\frac{3}{2}}(m)\left[\alpha A_\beta I_{\frac{3}{2}}(\beta) I_{\frac{1}{2}}(\alpha) - \beta A_\alpha I_{\frac{3}{2}}(\alpha) I_{\frac{1}{2}}(\beta)\right]} \tag{45}$$

$$D_n^{(1)} = \frac{1}{2}\frac{\begin{pmatrix}\begin{pmatrix}\begin{pmatrix}\begin{pmatrix}(1-n)(4n-2)(N-1)\Omega_4 Y_n m^2 + \alpha^2\beta^2 X_n \\ ((n-1)\Omega_1 + \Omega_2)(N-2)\end{pmatrix} mK_{n-\frac{1}{2}}(m) \\ +NX_n\Omega_5\alpha^2\beta^2 K_{n-\frac{3}{2}}(m)(N-2)\end{pmatrix} \\ A_\gamma I_{n-\frac{1}{2}}(\beta) + \alpha^2\beta^3 mK_{n-\frac{1}{2}}(m) A_\alpha X_n I_{n-\frac{3}{2}}(\beta)(N-2)\Omega_7 \end{pmatrix} I_{n-\frac{1}{2}}(\alpha) \\ -\alpha^3\beta^2 A_\beta mK_{n-\frac{1}{2}}(m) X_n I_{n-\frac{3}{2}}(\beta) I_{n-\frac{3}{2}}(\alpha)(N-2)\Omega_7\end{pmatrix} n}{\begin{pmatrix} A_\gamma \begin{pmatrix} -\left((4n^3+11n-12n^2-3)m^2+\alpha^2\beta^2 n\right) mK_{n-\frac{1}{2}}(m) \\ \alpha^2\beta^3(N-2)A_\alpha\left(n-\frac{3}{2}\right)mn\end{pmatrix} \\ I_{n-\frac{1}{2}}(\alpha) - K_{n-\frac{1}{2}}(m)\alpha^3 A_\beta\beta^2(N-2) I_{n-\frac{3}{2}}(\alpha)\left(n-\frac{3}{2}\right) mI_{n-\frac{1}{2}}(\beta) n\end{pmatrix}(N-2)}$$

where:

$$\Omega_1 = 2 - 2\,A_2^{(2)} - B_2^{(1)} + D_2^{(1)} - \frac{1}{2}\,D_2^{(2)}\,I_{\frac{3}{2}} + \frac{1}{2}\,E_2^{(1)}\,K_{\frac{3}{2}}(m) - \frac{1}{2}\,F_2^{(2)}\,I_{\frac{3}{2}}(\beta)$$

$$\Omega_2 = 2 + 2\,B_2^{(1)} + \tfrac{1}{2}\,E_2^{(1)}\left(K_{\frac{3}{2}}(m) - mK_{\frac{1}{2}}(m)\right) - 2\,A_2^{(2)} - \tfrac{1}{2}\,D_2^{(2)}\left(\alpha\,I_{\frac{1}{2}}(\alpha) + I_{\frac{3}{2}}(\alpha)\right) - \tfrac{1}{2}\,F_2^{(2)}\left(\beta\,I_{\frac{1}{2}}(\beta) + I_{\frac{3}{2}}(\beta)\right)$$

$$\Omega_3 = -6\,B_2^{(1)} + E_2^{(1)}\,K_{\frac{3}{2}}(m)\left(\frac{m^2}{2} - 3\right) - D_2^{(2)}\,I_{\frac{3}{2}}(\alpha)\left(\frac{\alpha^2}{2} - 3\right) - F_2^{(2)}\,I_{\frac{3}{2}}(\beta)\left(\frac{\beta^2}{2} - 3\right)$$

$$\Omega_4 = \frac{(2-N)}{(1-N)}\,D_2^{(1)} - \frac{(2-N)}{(1-N)}\left(\frac{\alpha^2\beta^2}{2m^2}\right)A_2^{(2)},$$

$$\Omega_5 = D_2^{(1)} + \left(\frac{m^2}{2N}\right)E_2^{(1)}\,K_{\frac{3}{2}}(m)$$

$$\Omega_6 = \frac{1}{2} D_2^{(2)} A_\alpha I_{\frac{3}{2}}(\alpha) + \frac{1}{2} F_2^{(2)} A_\beta I_{\frac{3}{2}}(\beta)$$

$$\Omega_7 = \mathrm{N}\Omega_5 - \Omega_3 + \Omega_1 \mathrm{n}(n - 1)$$

$$A_\gamma = \beta^2 A_\alpha - \alpha^2 A_\beta \tag{46}$$

The drag expression, (44), for creeping flow of a micropolar fluid past a porous approximate sphere can be simplified to that of the problem presented by Y. Qin and P.N. Kaloni in 1988 of *A Cartesian Tensor Solution of the Brinkman Equation.* When the found drag for the problem understudy, (44) – (46), is subjected to the conditions that the micropolar parameter tends to infinity, $m \to \infty$ and the coupling number, N, is marginalised, i.e., $N \to 0$, then $\alpha^2 \to \eta^2$, $\beta^2 \to \infty$, $A_\alpha \to \frac{\eta^2}{2}$, $A_\beta \to \infty$ and the drag expression reduces to:

$$D = \frac{\eta^2\,(sinh(\eta) - \eta\,cosh(\eta))}{\eta\,(3 + 2\,\eta^2)\,cosh(\eta) - 3\,sinh\,(\eta)} \tag{47}$$

which agrees with the drag on the porous sphere derived by Qin and Kaloni in 1988 when the fluid is Newtonian.

The drag expression for creeping flow of a micropolar fluid past a porous approximate sphere, given by (44), can be simplified to that of the problem presented by S.R. Majumdar and H. Ramkissoon in 1976, 'Axisymmetric Creeping Flow of a Micropolar Fluid past an Approximate Sphere'. When the found drag is subjected to the condition that the porosity parameter is marginalised, equations (44) – (46) reduce to:

$$D = \begin{array}{l} \left[(^r/_a)^2 + B_2(^a/_r) + D_2(^r/_a) + E_2(^r/_a)^{\frac{1}{2}} K_{3/_2}\left(^{\lambda r}/_a\right)\right] V_2(\varsigma) + \\ \left[B_{m-2}(^a/_r)^{m-3} + D_{m-2}(^a/_r)^{m-5} + E_{m-2}(^r/_a)^{\frac{1}{2}} K_{m-5/_2}\left(^{\lambda r}/_a\right)\right] V_{m-2}(\varsigma) + \\ \left[B_m(^a/_r)^{m-1} + D_m(^a/_r)^{m-3} + E_m(^r/_a)^{\frac{1}{2}} K_{m-1/_2}\left(^{\lambda r}/_a\right)\right] V_m(\varsigma) + \\ \left[B_{m+2}(^a/_r)^{m+1} + D_{m+2}(^a/_r)^{m-1} + E_{m+2}(^r/_a)^{\frac{1}{2}} K_{m+5/_2}\left(^{\lambda r}/_a\right)\right] V_{m+2}(\varsigma) + \end{array} \tag{48}$$

which agrees with the drag on the approximate sphere derived by Majumdar and Ramkissoon in 1976 (Ramkissoon, 1976).

VII. NUMERICAL ANALYSIS

For the sake of simplicity, let the micropolar drag be represented in each plot by D_N. The drag formula, (44), was plotted against its non-porous and Newtonian counterparts with permeability parameter η^2 for $m = 20$ (micropolar coupling number) and for the value of $N = 0.1$, as shown in Figure 1.

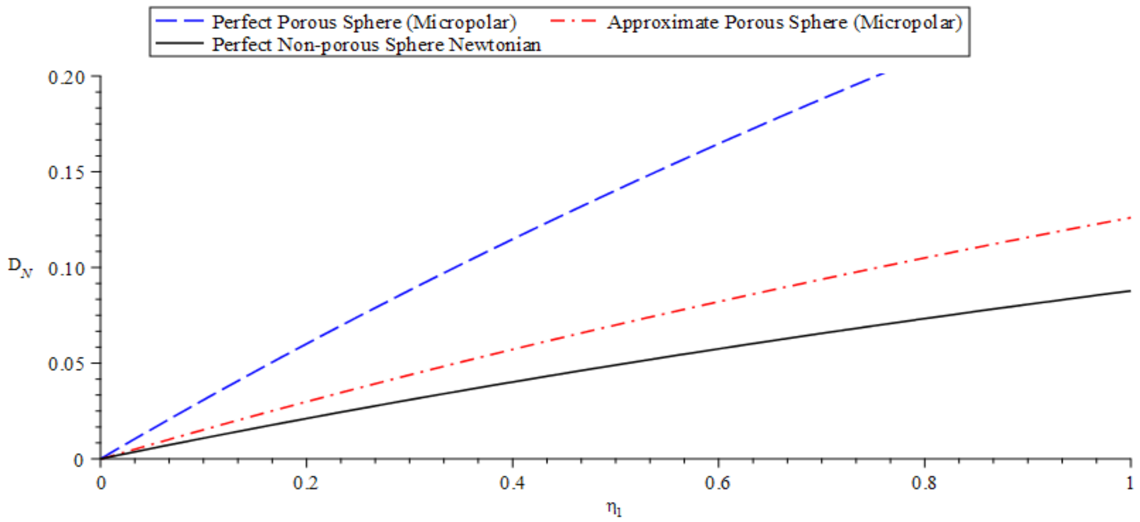

Figure 1: Graph showing the plot of an Approximate Sphere vs. a perfect Porous Sphere when $N = 0.1$

It can be observed that when coupling number $N = 0.1$, the drag of a creeping micropolar fluid past a porous approximate sphere is less than that of its non-porous counterpart and greater than that of the classical Newtonian case. The drag formula, (44), was plotted against its non-porous and Newtonian counterparts with permeability parameter η^2 for $m = 20$ (micropolar coupling number) and for the value of $N = 0.25$ as shown in Figure 2.

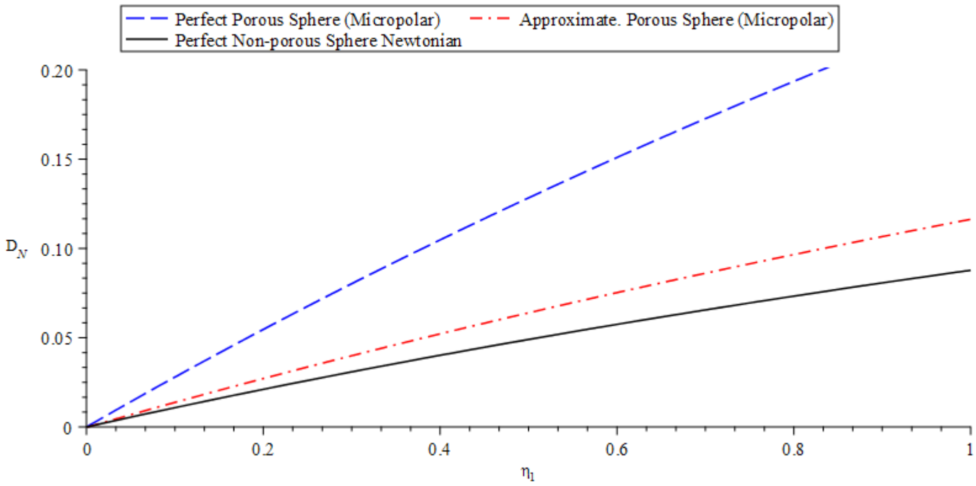

Figure 2: Graph showing the plot of an Approximate Sphere vs. a perfect Porous Sphere when $N = 0.25$

It can be observed that when coupling number $N = 0.25$, the drag of a creeping micropolar fluid past a porous approximate sphere is lesser than that of its non-porous counterpart and greater than that of the classical Newtonian case.

The drag formula, (44), was plotted against its non-porous and Newtonian counterparts with permeability parameter η^2 for $m = 20$, (micropolar coupling number), and for the value of $N = 0.5$, as shown in Figure 3.

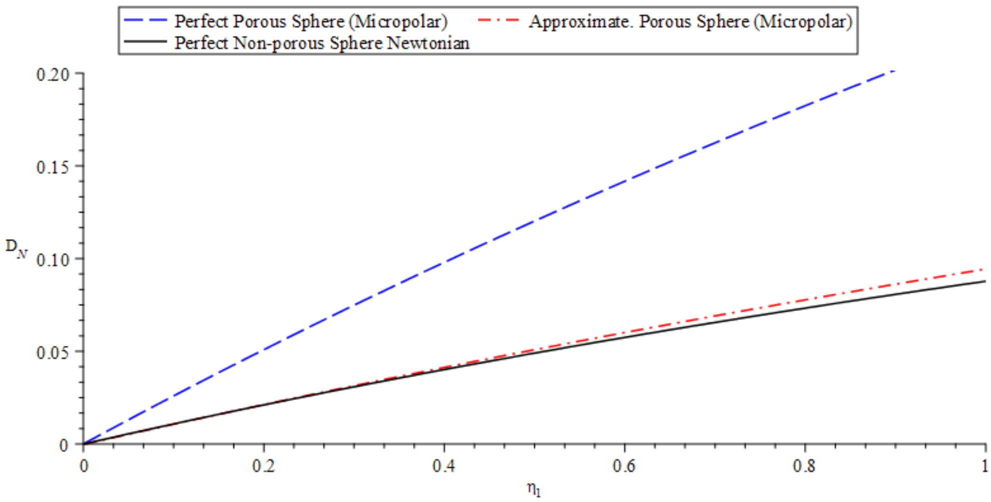

Figure 3: Graph showing the plot of an Approximate Sphere vs. a perfect Porous Sphere when $N = 0.5$

It can be observed that when coupling number $N = 0.5$, the drag of a creeping micropolar fluid past a porous approximate sphere is lesser than that of its non-porous counterpart and greater than that of the classical Newtonian case.

The drag formula, (44), was plotted against its non-porous and Newtonian counterparts with permeability parameter η^2 for $N = 0.5$ (micropolar coupling number) and for the value of $m = 2$ shown in Figure 4.

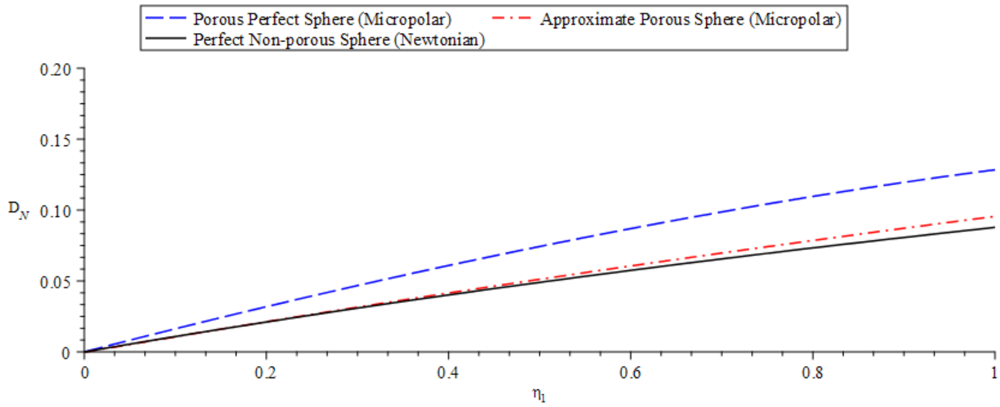

Figure 4: Graph showing the plot of an Approximate Sphere vs. a perfect Porous Sphere when $m = 2$

It can be observed that when coupling number $m = 2$, the drag of a creeping micropolar fluid past a porous approximate sphere is lesser than that of its non-porous counterpart and greater than that of the classical Newtonian case.

The drag formula, (44), was plotted against its non-porous and Newtonian counterparts with permeability parameter η^2 for $N = 0.5$, (micropolar coupling number), and for the value of $m = 10$ as shown in Figure 5.

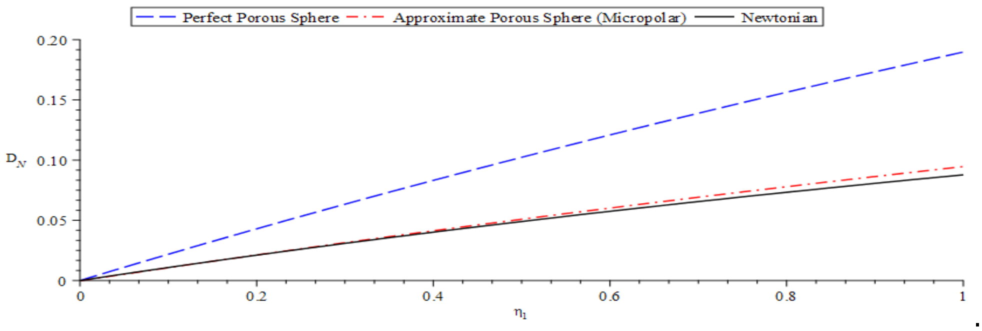

Figure 5: Graph showing the plot of an Approximate Sphere vs. a Perfect Porous Sphere when $m = 10$

It can be observed that when coupling number $m = 10$, the drag of a creeping micropolar fluid past a porous approximate sphere is less than that of its non-porous counterpart and greater than that of the classical Newtonian case.

The drag formula, (44), was plotted against its non-porous and Newtonian counterparts with permeability parameter η^2 for $N = 0.5$, (micropolar coupling number), and for the value of $m = 50$ as shown in Figure 6.

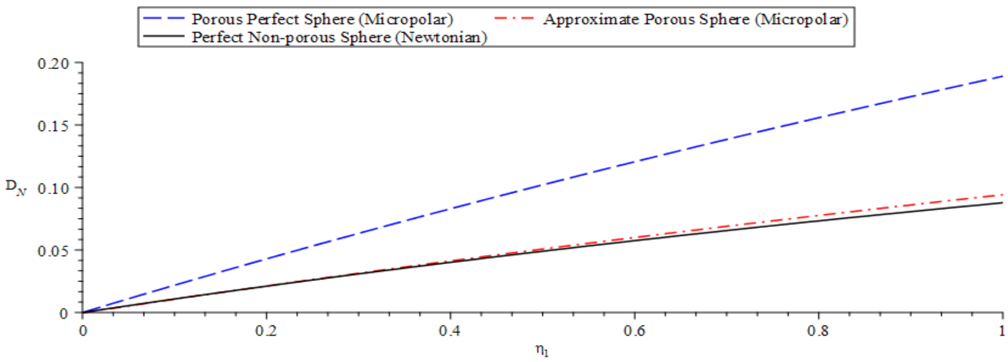

Figure 6: Graph showing the plot of an Approximate Sphere vs. a perfect Porous Sphere when $m = 50$

It can be observed that when coupling number $m = 50$, the drag of a creeping micropolar fluid past a porous approximate sphere is less than that of its non-porous counterpart and greater than that of the classical Newtonian case.

The final plot shows the variation of the drag formula, (44), for creeping flow of a micropolar fluid past a porous approximate sphere with permeability parameter η^2 for $N = 0.5$ and various values of *m.* It can be observed from this plot that as the coupling number m increases, the drag decreases. This is in accordance with micropolar fluid theory as it is known that a micropolar fluid reduces to that of the classical Newtonian case when coupling number N tends towards zero and coupling number m tends towards infinity. This implicit plot is given in Figure (7).

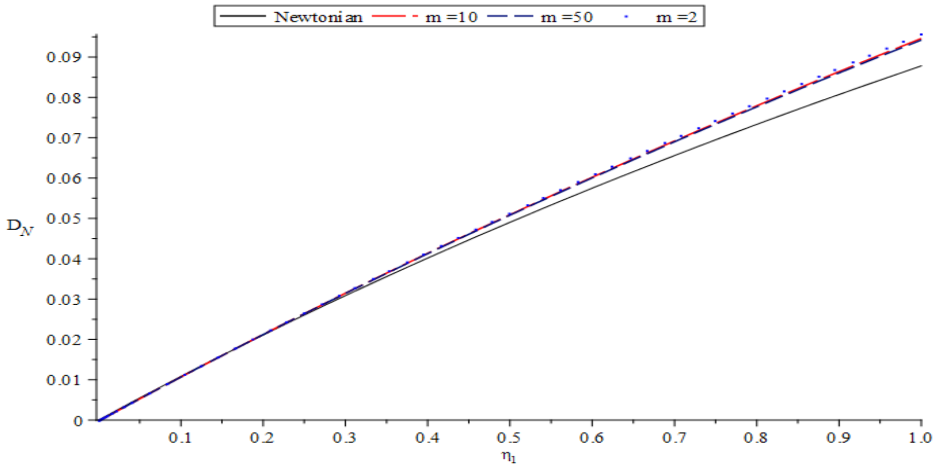

Figure 7: Implicit plot of Drag showing the Approximate Porous Sphere with various values of *m*.

VIII. CONCLUSION

Upon observation of the resultant drag formulae for creeping micropolar flow past a porous approximate sphere, the following observations were observed as central characteristic results.

1. As permeability increases, the drag on the porous approximate sphere decreases, when the fluid is micropolar.
2. As the coupling number N is decreasing, the drag on the porous approximate sphere also decreases, when the fluid is micropolar.
3. The drag on the porous approximate sphere, when the fluid is micropolar, is always greater than that of the known Newtonian case.
4. As the coupling number m decreases, the drag on the porous approximate sphere also decreases, when the fluid is micropolar.
5. Only β_3 and β_5 contribute to the drag on the approximate sphere in a creeping micropolar fluid. This implies that in this case, the drag is relatively insensitive to the details of the surface geometry.
6. The drag on the porous sphere is more than that of the drag on the porous approximate sphere, when the fluid is micropolar.

IX. REFERENCES

[1] Eringen, A. (1964). Simple Microfluids. *International Journal of Engineering Science, 2*, 205-217.

[2] Eringen, A. (1966). Theory of Micropolar Fluids. *Journal of Mathematics and Mechanics, 16*, 1-18.

[3] Iyengar, T., & Srinivasacharya, D. (1995). Stokes Flow of an Incompressible Micropolar Fluid Past an Approximate Sphere. *International Journal of Engineering Science, 31*(1), 115-123.

[4] Majmudar, S., & Ramkissoon, H. (1977, June). Axisymmetric Creeping Flow of a Micropolar Fluid. *the Tensor (New Series), 31*(2), 107-117.

[5] Neale, G., Epstein, N., & Nader, W. (1973). Creeping Flow Relative to Permeable Spheres. *Chemical Engineering Science, 28*, 1865-1874.

[6] Ramkissoon, H. (1997). Polar Flow Past a Spheroid. *Engineering Transactions, 45*, 171-180.

[7] Ramkissoon, H. (1997). Slip Flow past an Approximate Spheroid. *Acta Mechanica, 123*, 227-233.

[8] Ramkissoon, H., & Majumdar, S. (1975). Creeping Flow of a Micropolar Fluid Past a Sphere. *Letters in Applied and Engineering Sciences, 3*, 133-142.

[9] Srinivasacharya, D. (2003). Creeping Flow Past a Porous Approximate Sphere. *Zeitschrift für Angewandte Applied Mathematics and Mechanics, 83*(7), 499-504.

[10] Srinivasacharya, D., & Rajyalakshmi, I. (2004). Creeping Flow of Micropolar Fluid Past a Porous Sphere. *Applied Mathematics and Computation, 153*, 843-854.